Photonic crystals: theory and applications

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Alexander Petrov
Technische Universität Hamburg-Harburg

SUPERVISOR

Prof. M. Eich

COWORKERS

G. Böttger
K. Preusser-Mellert
M. Schmidt

ACKNOWLEDGEMENTS

- Steven G. Johnson for some diagrams from his photonic crystal tutorial and for using his MIT-MPB PW software
- CST Darmstadt for supplying us with their MWS Software

INTRODUCTORY BOOKS

- K. Sakoda, Optical Properties of Photonic Crystals, Springer 2001 Joannopoulos
- S.G. Johnson, J.D. Joannopoulos, Photonic Crystals: The Road from Theory to Practice, Kluwer 2002
- J.D. Joannopulos et al., Photonic Crystals, Princeton Univ. Press 1995

Theory of infinite PC structure

Beam propagation in PC

PC as omnidirectional mirror

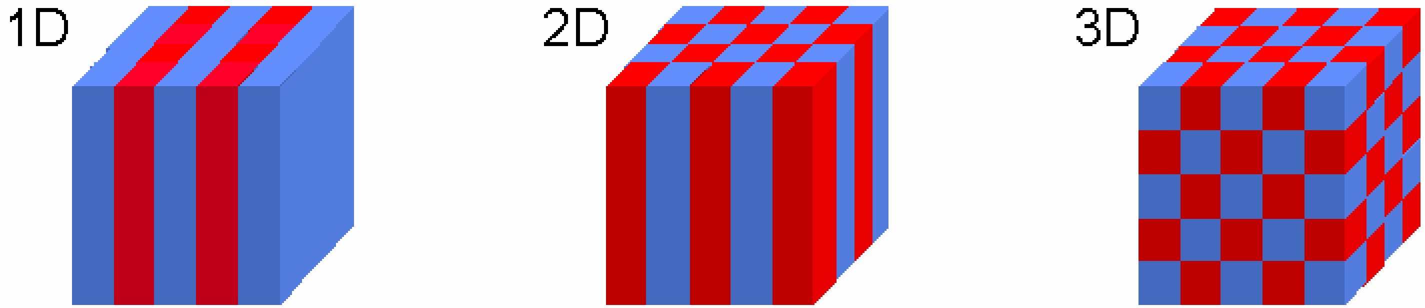
2D PC slab structure

Manufacturing

Possible applications

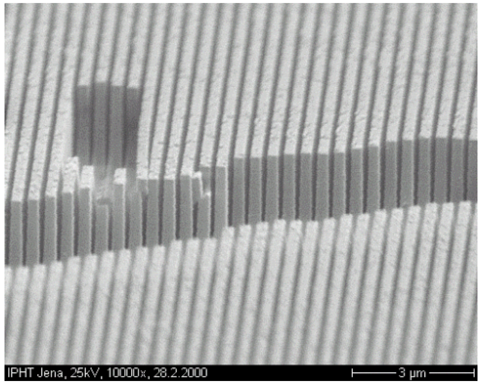
Photonic crystal is a periodical dielectric material

EXAMPLES OF PHOTONIC CRYSTALS

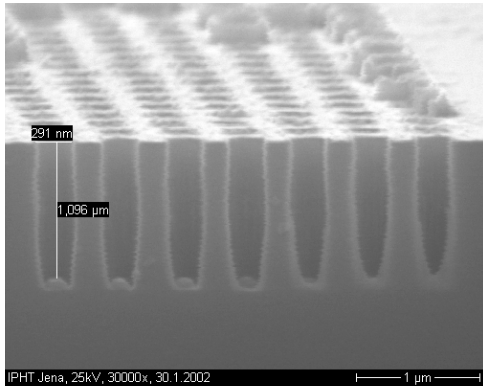


[Joannopoulos et al., „Photonic Crystals , Molding the Flow of Light“ (1995)]

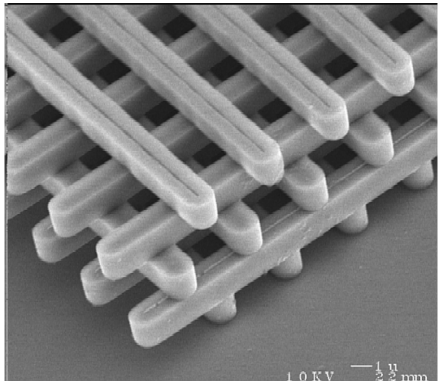
Lattice constant a ~ λ



[Meyer et al., IPHT, Jena]



[Liguda, Eich et al., TU-Hamburg]



[Lin et al., Sandia, New Mexico]

Maxwell's equations rewritten to an eigenvalue problem

MAXWELL'S EQUATIONS

$$\begin{cases} \nabla \cdot \{\varepsilon(\vec{r})\,\vec{E}(\vec{r},t)\} = 0 \\ \nabla \cdot \vec{H}(\vec{r},t) = 0 \\ \nabla \times \vec{E}(\vec{r},t) = -\mu_0 \dfrac{\partial \vec{H}(\vec{r},t)}{\partial t} \\ \nabla \times \vec{H}(\vec{r},t) = \varepsilon_0 \varepsilon(\vec{r}) \dfrac{\partial \vec{E}(\vec{r},t)}{\partial t} \end{cases}$$

WAVE EQUATIONS

$$\nabla \times \left\{ \frac{1}{\varepsilon(\vec{r})} \nabla \times \vec{H}(\vec{r},t) \right\} = -\frac{1}{c^2} \frac{\partial^2 \vec{H}(\vec{r},t)}{\partial^2 t}$$

$$\frac{1}{\varepsilon(\vec{r})} \nabla \times \left\{ \nabla \times \vec{E}(\vec{r},t) \right\} = -\frac{1}{c^2} \frac{\partial^2 \vec{E}(\vec{r},t)}{\partial^2 t}$$

$$\vec{E} = \vec{E}(\vec{r}) \exp(-i\omega t)$$

$$\vec{H} = \vec{H}(\vec{r}) \exp(-i\omega t)$$

EIGENVALUE PROBLEM

$$L_E\, \vec{E}(\vec{r}) = \frac{\omega^2}{c^2} \vec{E}(\vec{r})$$

$$L_H\, \vec{H}(\vec{r}) = \frac{\omega^2}{c^2} \vec{H}(\vec{r})$$

Spatial periodicity allows the use of Fourier expansion

AN APPROACH TO SOLVE THE EIGENVALUE PROBLEM

$$L_E \mathbf{E}(\mathbf{r}) \equiv \frac{1}{\varepsilon(\mathbf{r})} \nabla \times \{\nabla \times \mathbf{E}(\mathbf{r})\} = \frac{\omega^2}{c^2} \mathbf{E}(\mathbf{r})$$

eigenvalue problem

$$\varepsilon(\mathbf{r}+\mathbf{R}) = \varepsilon(\mathbf{r})$$

$$\mathbf{R} = m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + m_3\mathbf{a}_3$$

periodicity

Bloch theorem

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_{\mathbf{k}n}(\mathbf{r}) = \mathbf{u}_{\mathbf{k}n}(\mathbf{r})\mathbf{e}^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\mathbf{u}_{\mathbf{k}n}(\mathbf{r}+\mathbf{R}) = \mathbf{u}_{\mathbf{k}n}(\mathbf{r})$$

Fourier expansion

$$\mathbf{E}_{\mathbf{k}n}(\mathbf{r}) = \sum_{\mathbf{K}} \mathbf{E}_{\mathbf{k}n}(\mathbf{K}) \exp\{i(\mathbf{k}+\mathbf{K})\cdot\mathbf{r}\}$$

$$\frac{1}{\varepsilon(\mathbf{r})} = \sum_{\mathbf{K}} e(\mathbf{K}) \exp(i\mathbf{K}\cdot\mathbf{r})$$

reciprocal coordinate system

$$\mathbf{b}_i = \frac{2\pi(\mathbf{a}_j \times \mathbf{a}_k)}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}, \qquad (ijk) = (123), (231), (312)$$

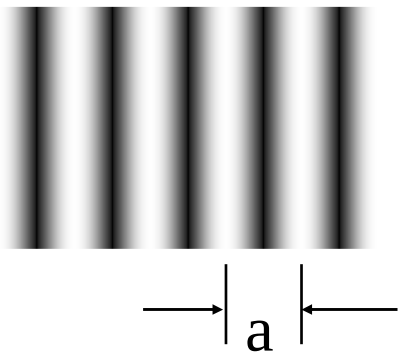
$$\mathbf{K} = l_1\mathbf{b}_1 + l_2\mathbf{b}_2 + l_3\mathbf{b}_3$$

Periodical dielectric function couples the spatial harmonics of electromagnetic field

EXAMPLE OF 1D PHOTONIC CRYSTAL

$$\varepsilon = \varepsilon_0[1 + M\cos(\vec{K}\,\vec{r})] \qquad K = \frac{2\pi}{a}$$

$$\Delta\vec{E} + k^2\varepsilon E = 0 \quad , \qquad \text{wave equation} \qquad k = \frac{\omega}{c}$$



Floquet-Bloch Wave:

$$\vec{E} = \vec{e}_z E = \vec{e}_z \sum_{n=-\infty}^{\infty} V_n \exp(i\vec{k}_n \cdot \vec{r}), \qquad \vec{k}_n = \vec{k}_0 + n\vec{K}$$

$$q(V_n) = (k^2 - \vec{k}_n^2)V_n + (M/2)k^2\{V_{n-1} + V_{n+1}\} = 0, \qquad -\infty < n < +\infty$$

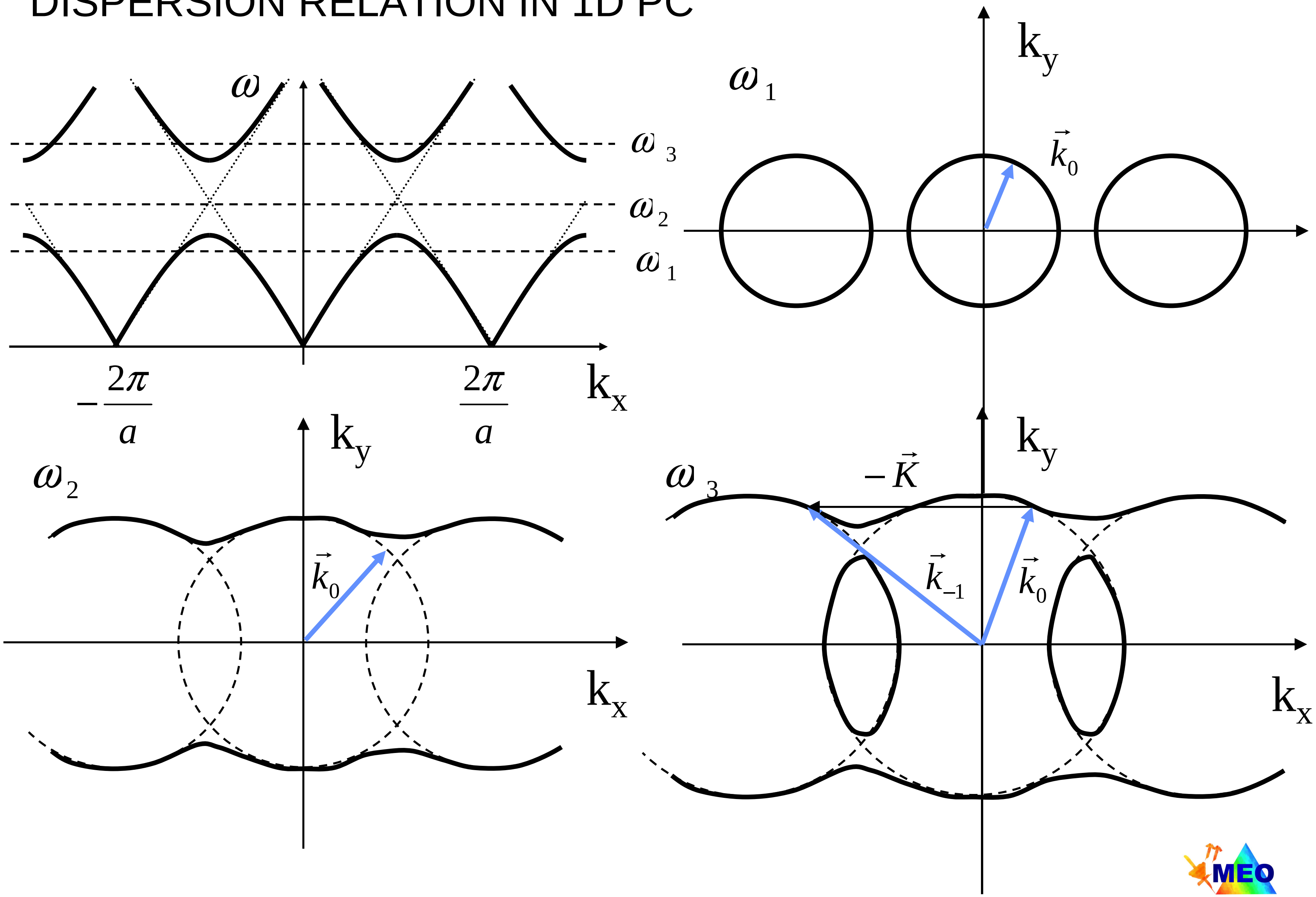
Setting the determinant of the coefficient matrix to zero leads to the dispersion relation:

$$\vec{k}_n(k) \equiv \vec{k}_0(k) + n\vec{K}$$

P.Russell Appl.Phys.B 39

Local band gap appears at the anti-crossing point

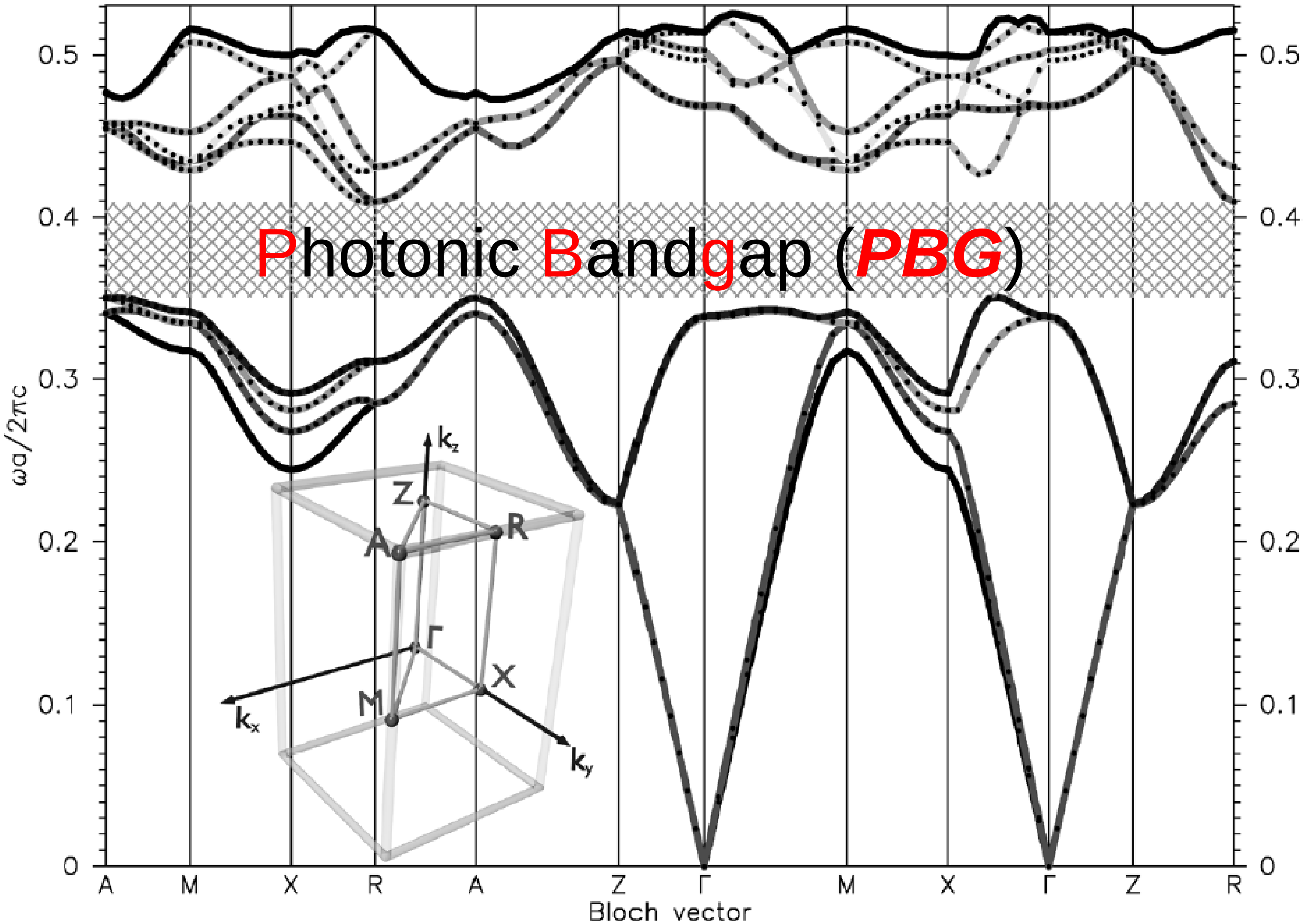
DISPERSION RELATION IN 1D PC



Omni-directional band gap in 3D PC structure

BAND DIAGRAM

[Toader et al., Science 292, 2001]



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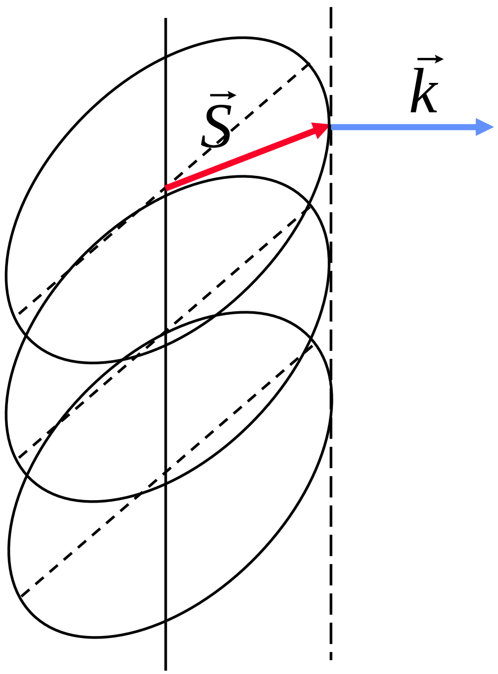
2D PC slab structure

Manufacturing

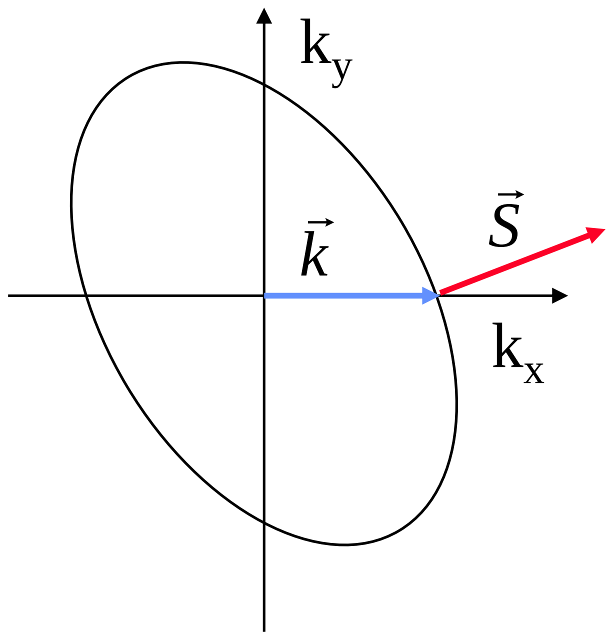
Possible applications

Light beam propagates with the group velocity

EXAMPLE OF BIREFRINGENT CRYSTAL



Real space
Huygens approach



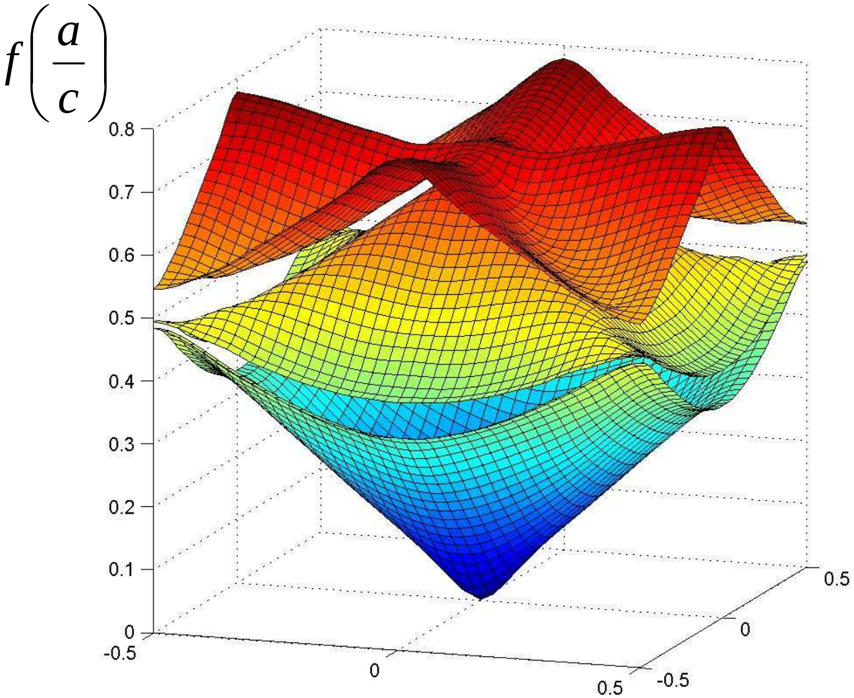
Reciprocal space
Wave vector diagram

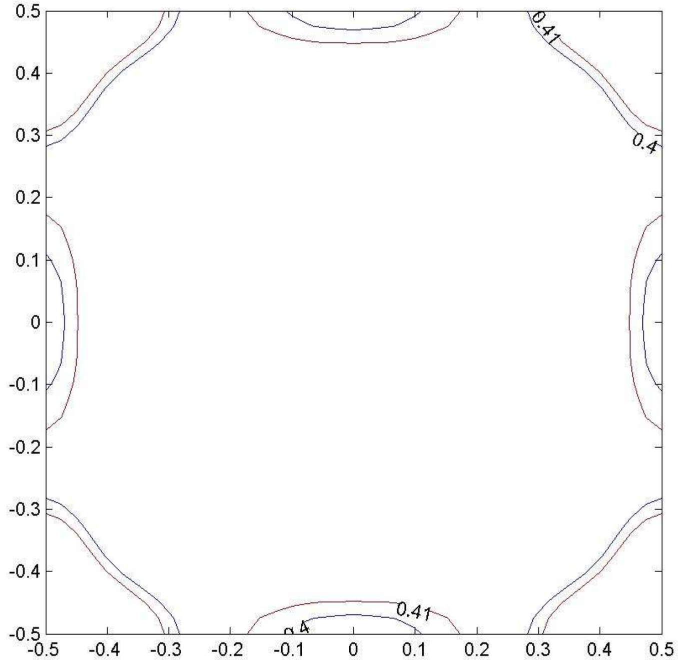
$$\vec{c}_g = \nabla_k \omega\, (\vec{k})$$

Mathematical representation

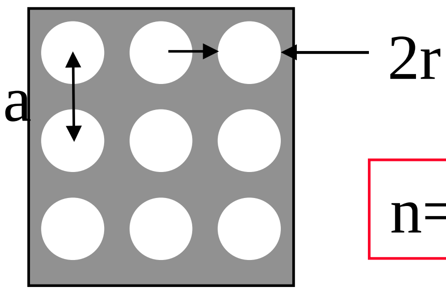
Dispersion relation of PCs can be quite complex

EXAMPLE OF 2D PC DISPERSION DIAGRAM





First Brillouin zone

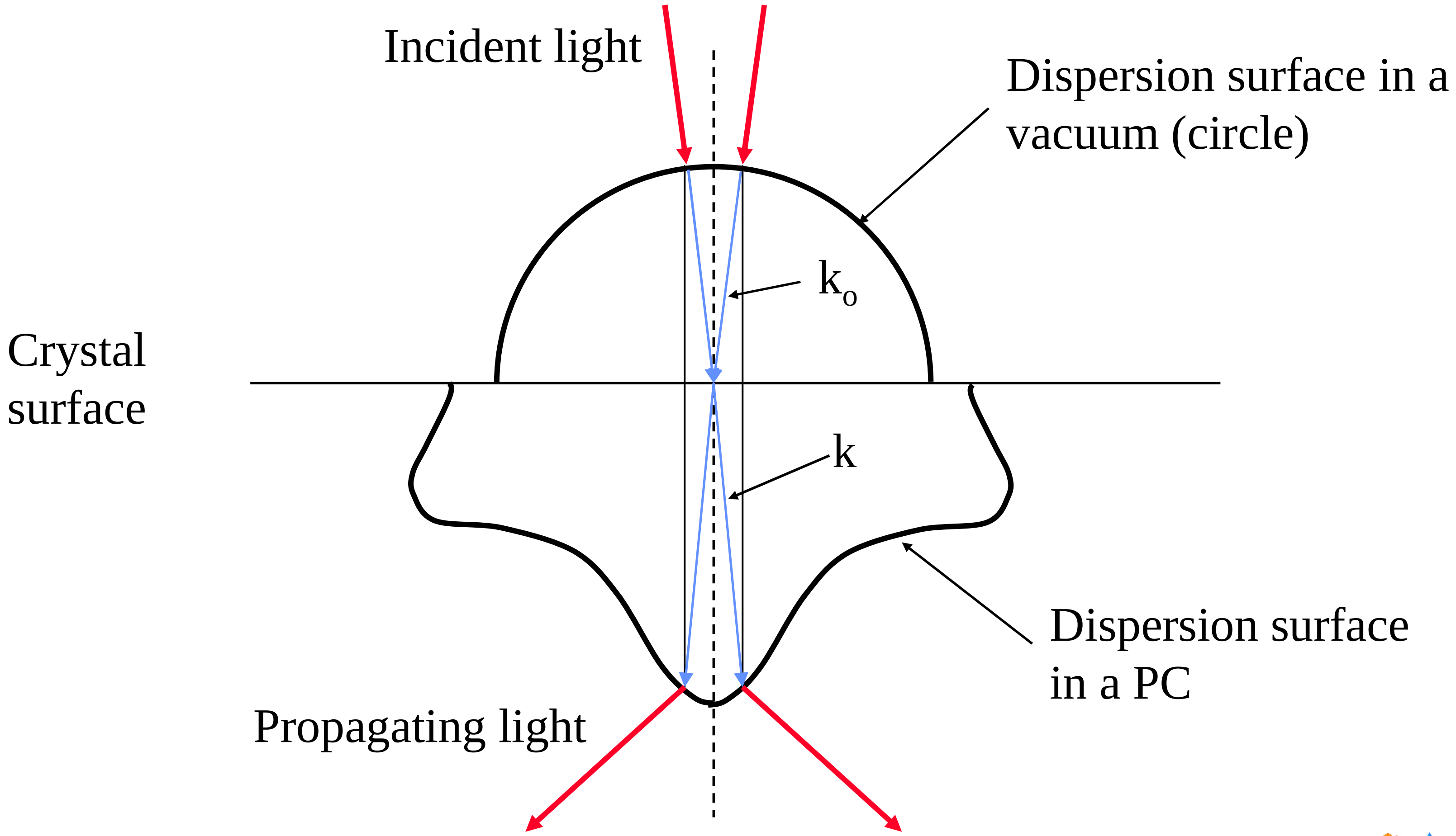


n=1.54

$$f\left(\frac{a}{c}\right) = 0.4, 0.41$$

Snell's law can be applied at the PC interface

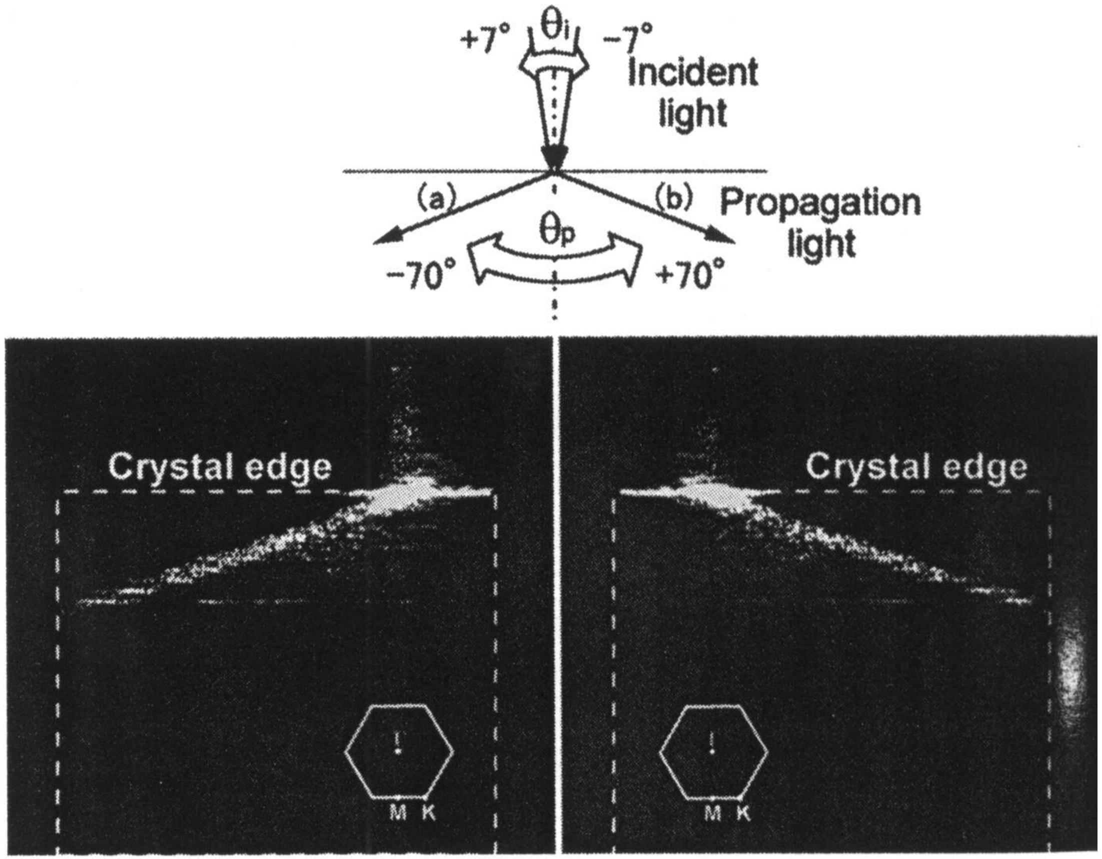
SCHEMATIC WAVE VECTOR DIAGRAM



H.Kosaka et al. Phys.Rev.B 62

Ultra-refractive phenomena can be demonstrated

EXPERIMENT



H.Kosaka et al. Phys.Rev.B 62

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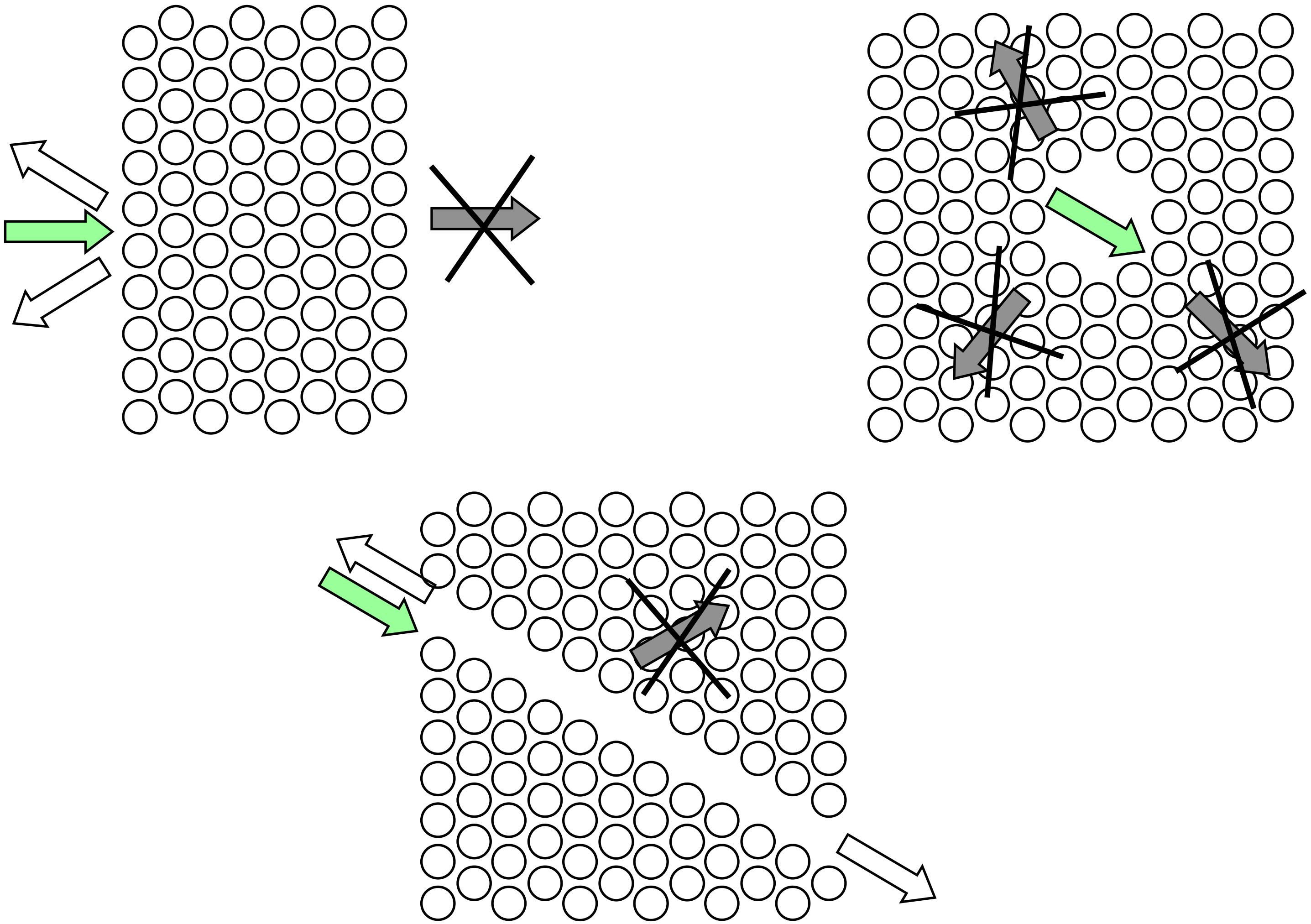
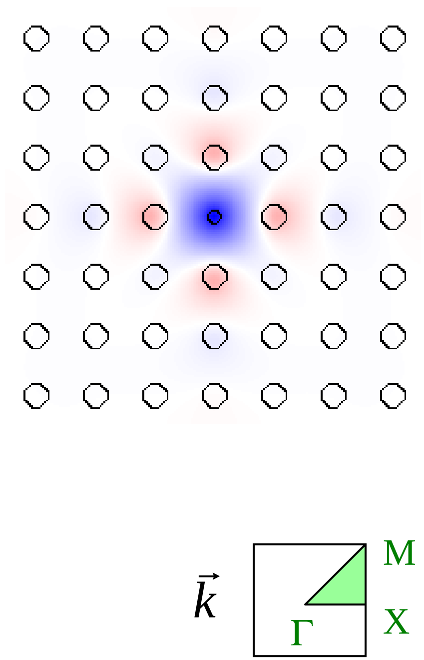
PC is an omni directional reflector at PBG frequencies

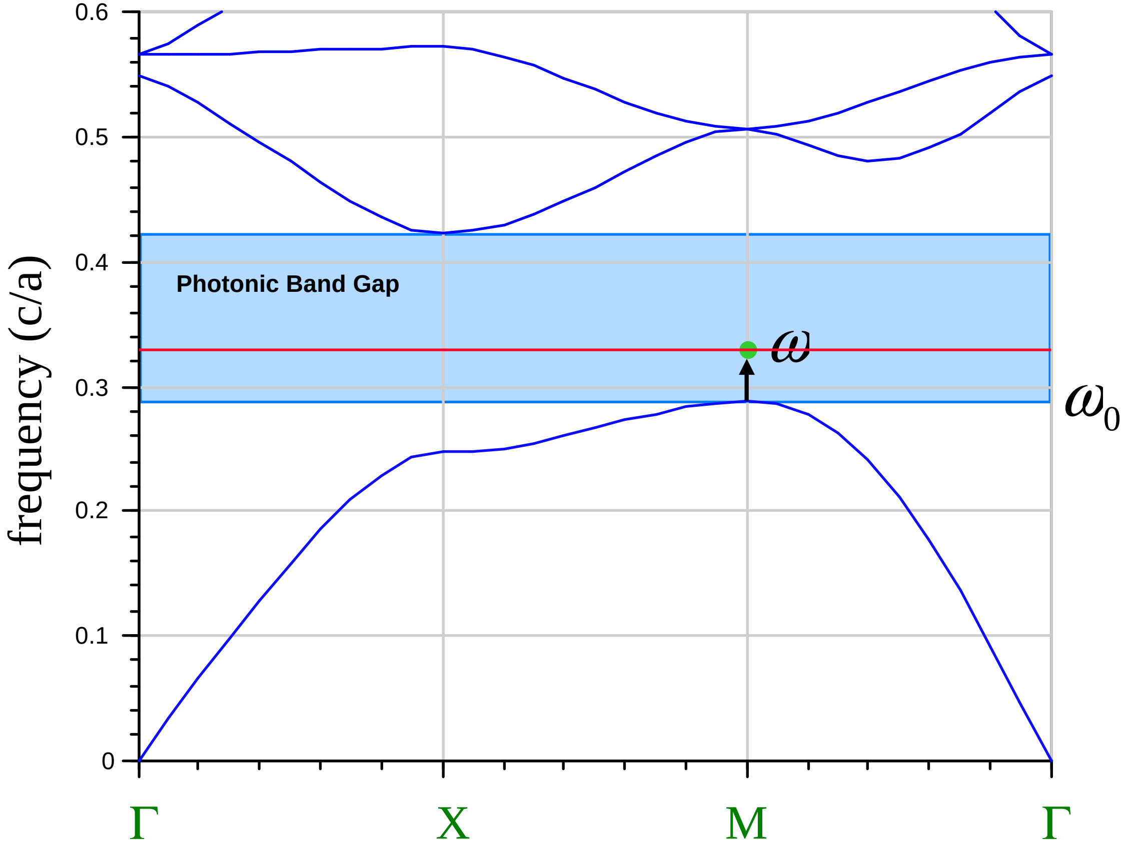
OMNIDIRECTIONAL MIRROR, CAVITY AND WAVEGUIDE



Defect creates a mode inside PBG region

AIR DEFECT MODE FROM REDUCED ROD SIZE

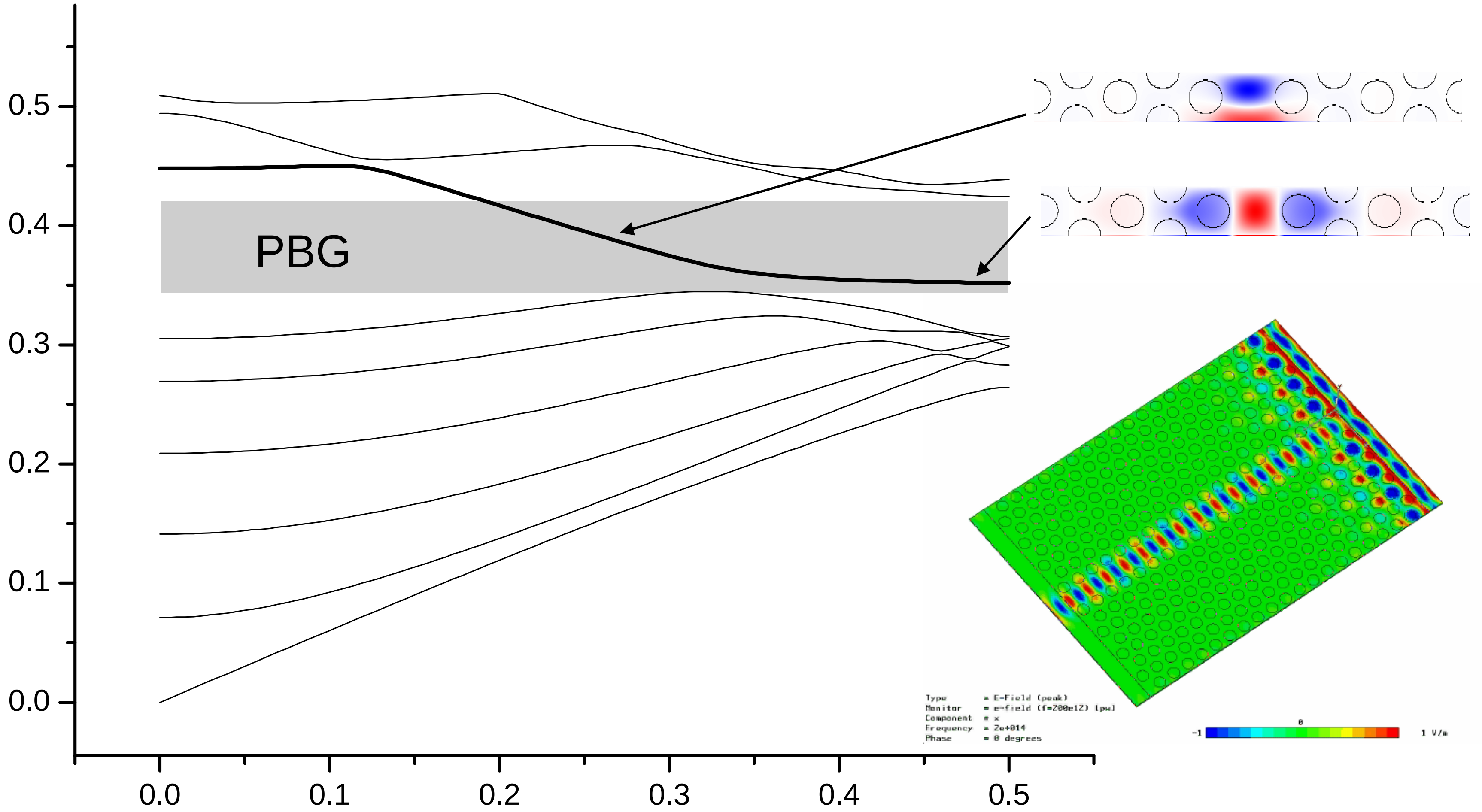




S. Johnson, tutorial , MIT

Line defect allows modes propagating along the defect

BAND DIAGRAM OF A PHOTONIC CRYSTAL WAVEGUIDE



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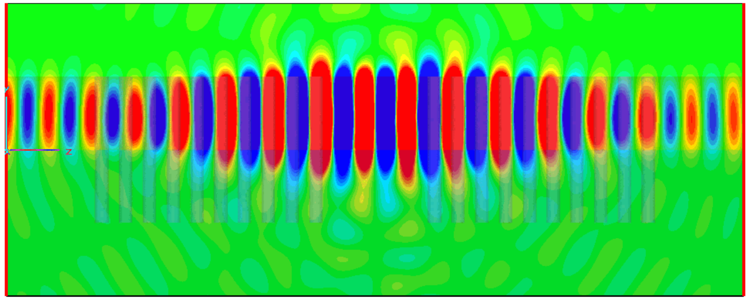
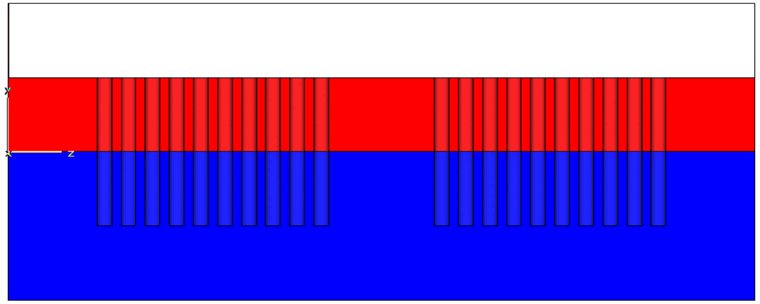
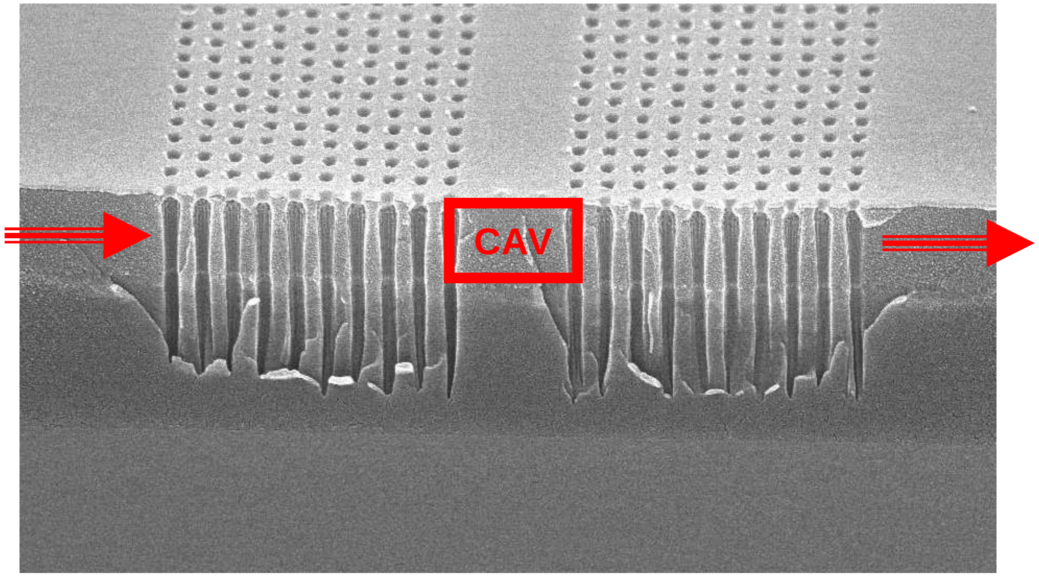
Some applications don't nee a complete 3D PBG

2D PC RESONATOR IN THE SLAB WAVEGUIDE



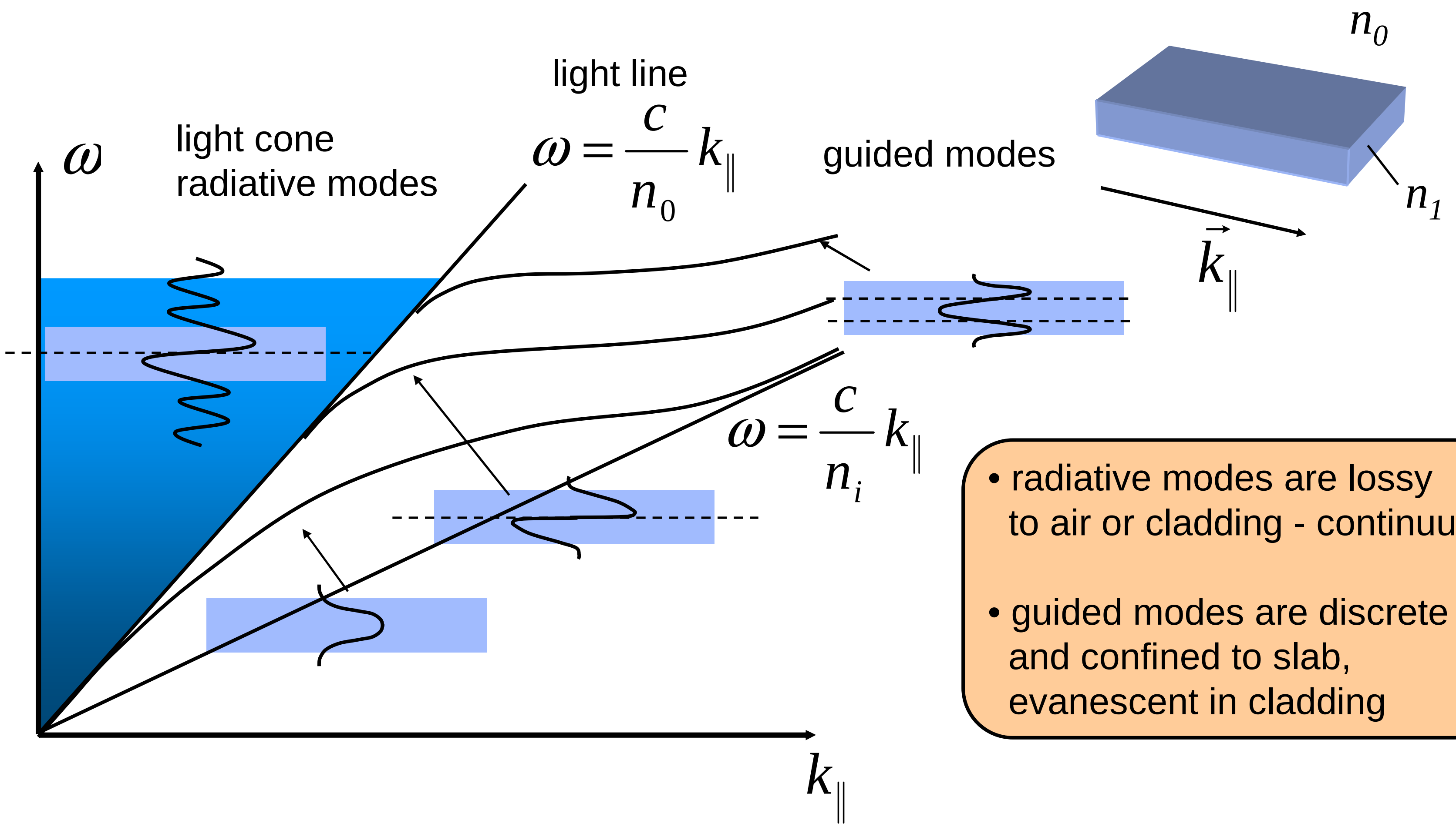
Q=1076





Only modes below light line are guided in slab waveguide

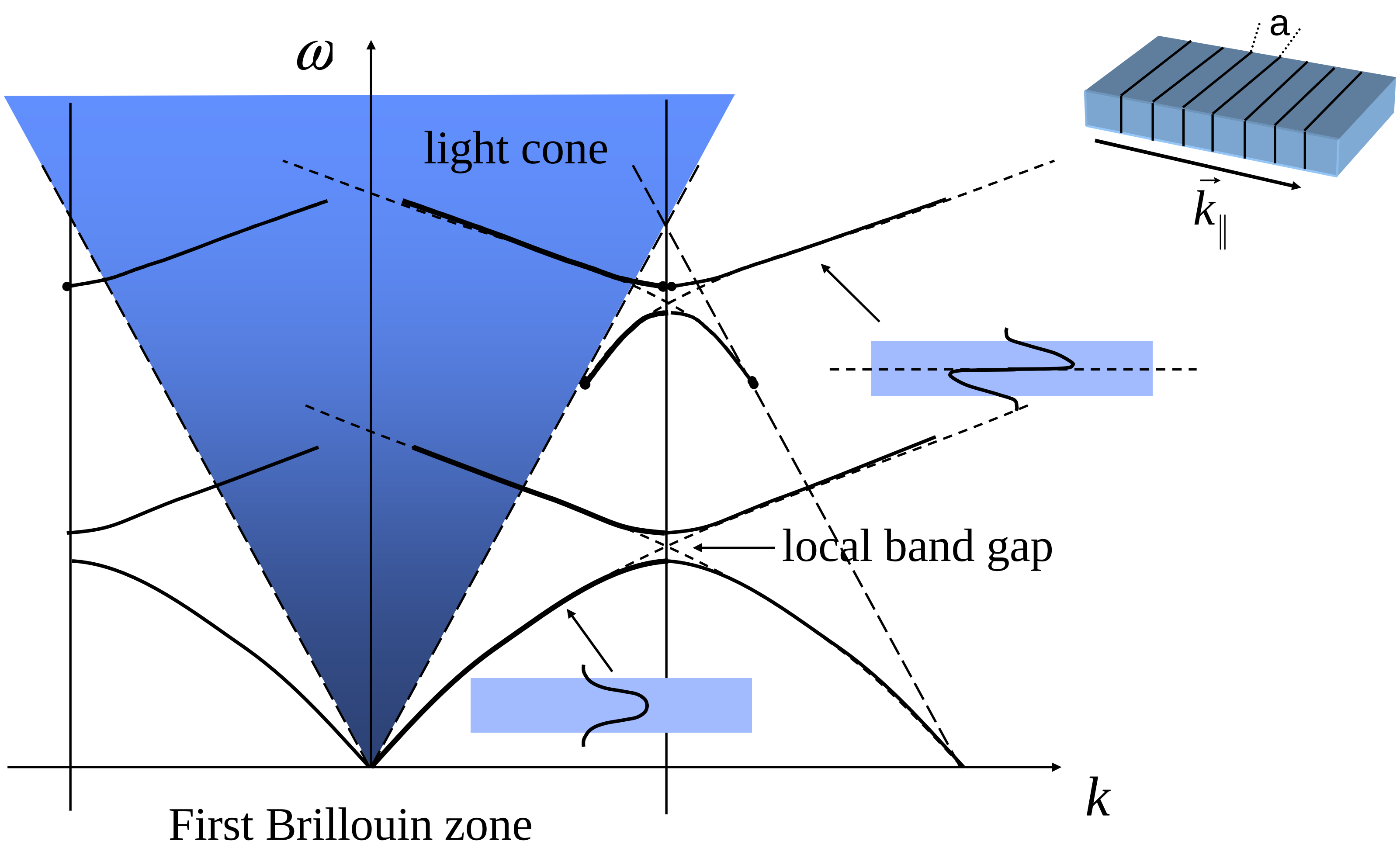
DISPERSION RELATION OF DIELECTRIC SLAB WAVEGUIDE



- radiative modes are lossy to air or cladding - continuum
- guided modes are discrete and confined to slab, evanescent in cladding

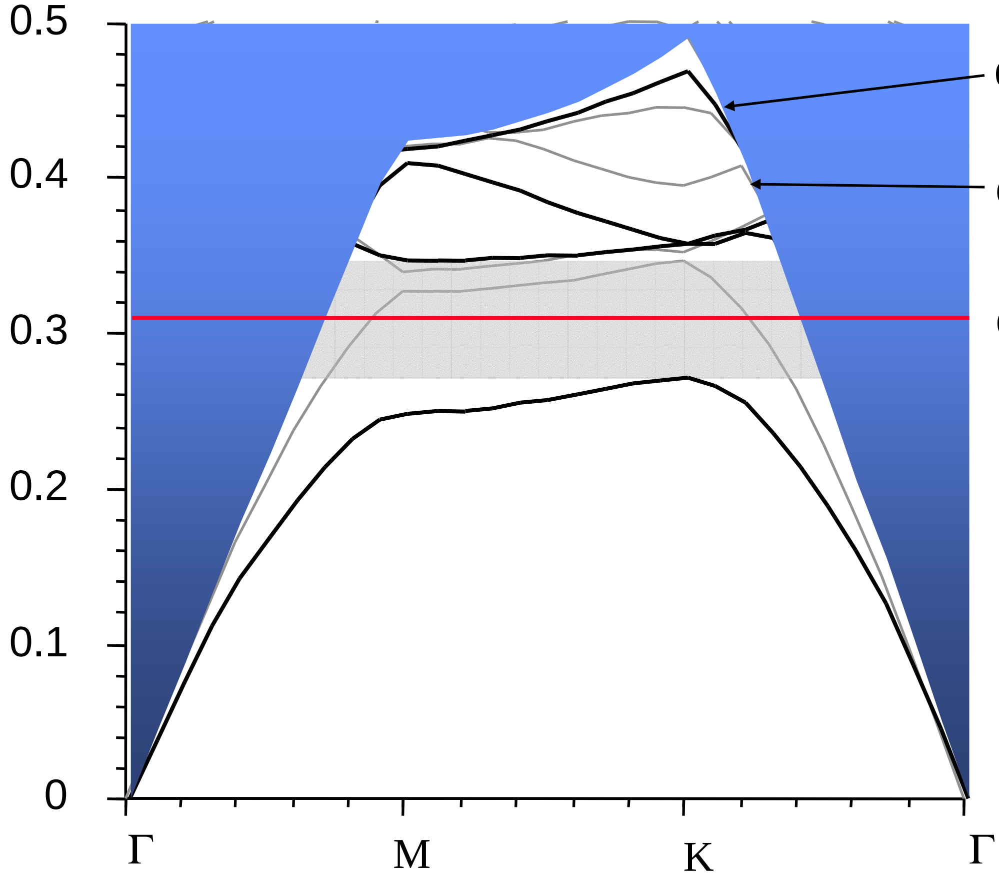
Modes under light line can have local band gap

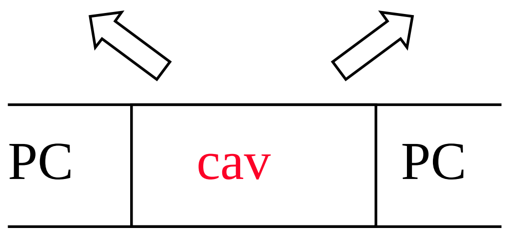
1D PC SLAB BAND DIAGRAM



Cavity in the 2D PC slab has intrinsic vertical losses

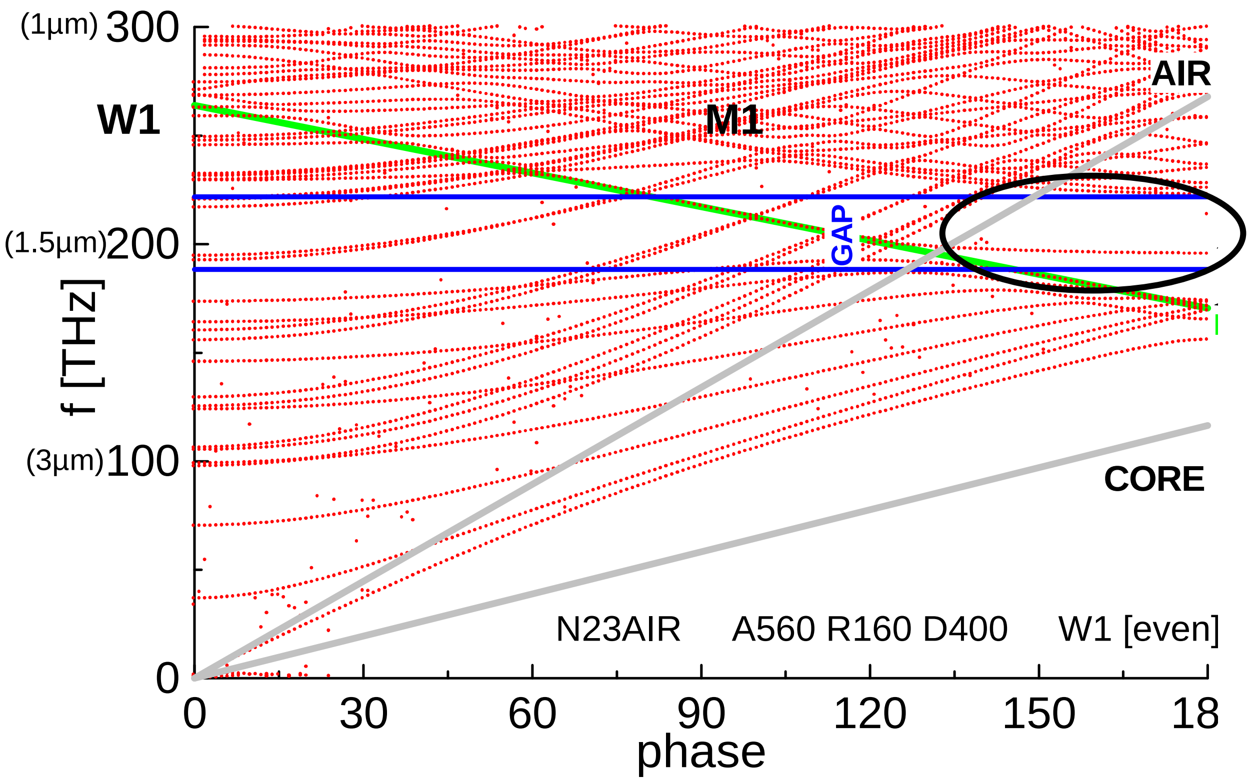
BAND DIAGRAM OF A DEFECT IN 2D SLAB PC

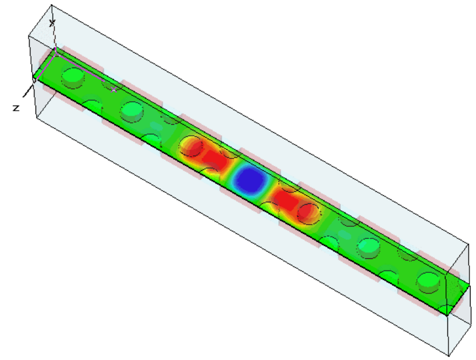


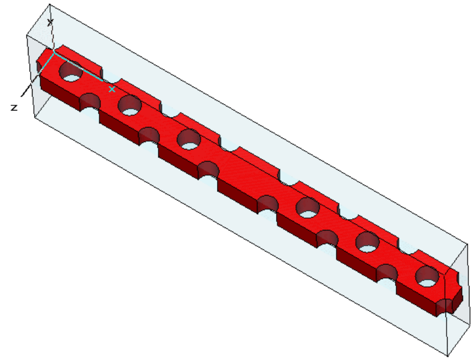


The useful bandwidth of the PC waveguide is reduced

BAND DIAGRAM OF PC SLAB WAVEGUIDE (AIR-BRIDGE)

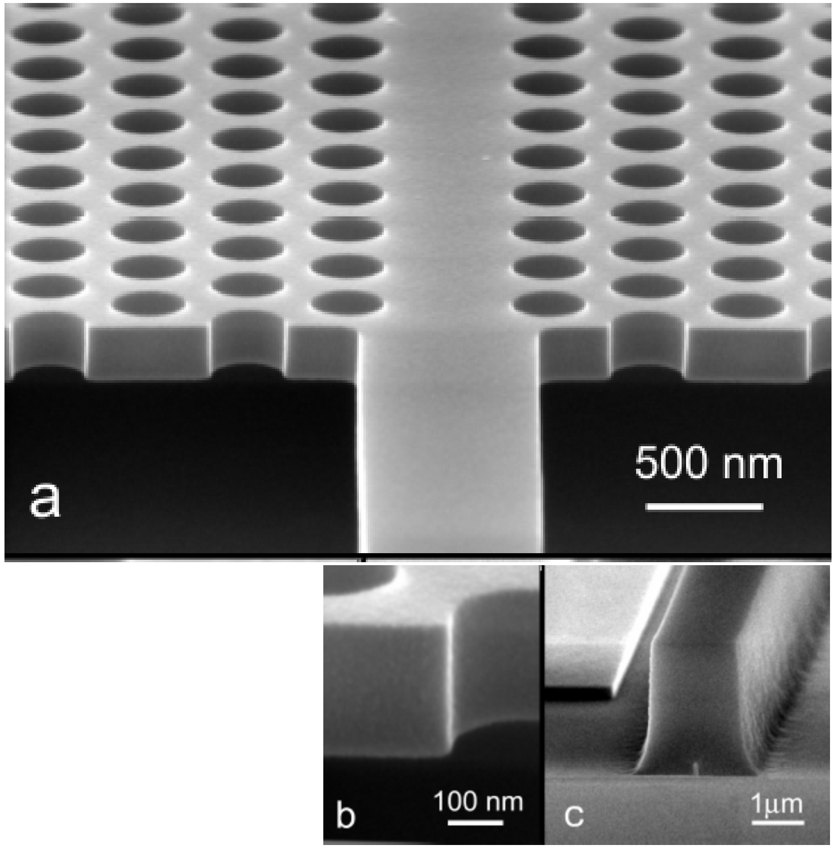


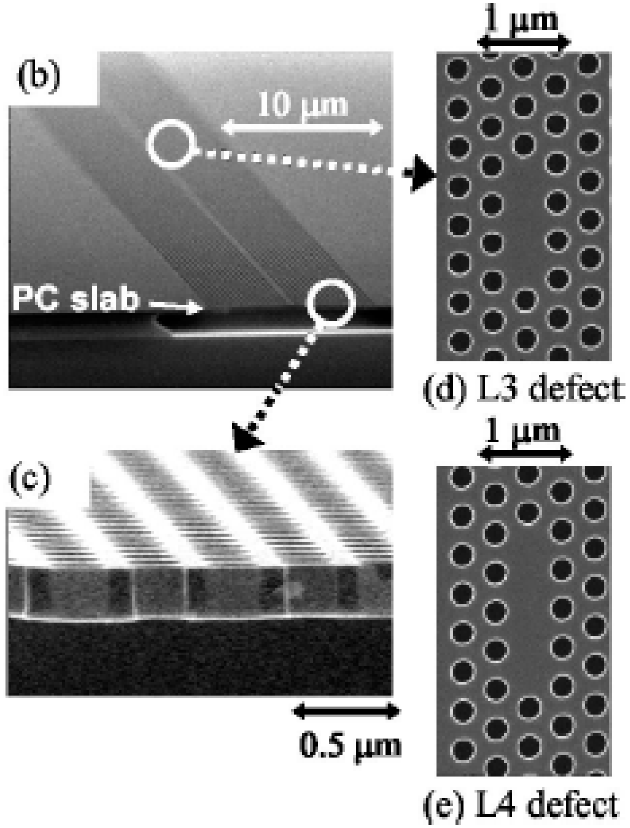




All the advantages of lithography technology are in favor of 2D PC slab structures

EXAMPLES OF 2D PC STRUCTURES





McNab et. al. Opt.Expr. 11; Akahane et. al. APL 83

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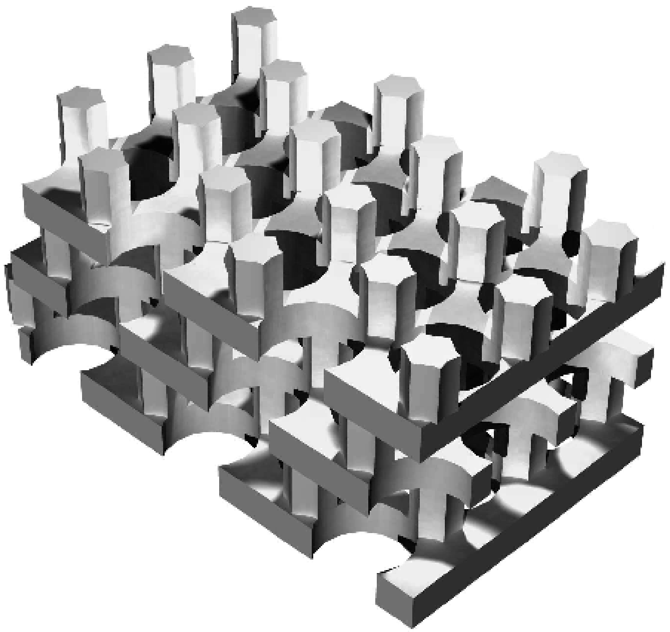
Possible applications

Different approaches are developed for 3D PC

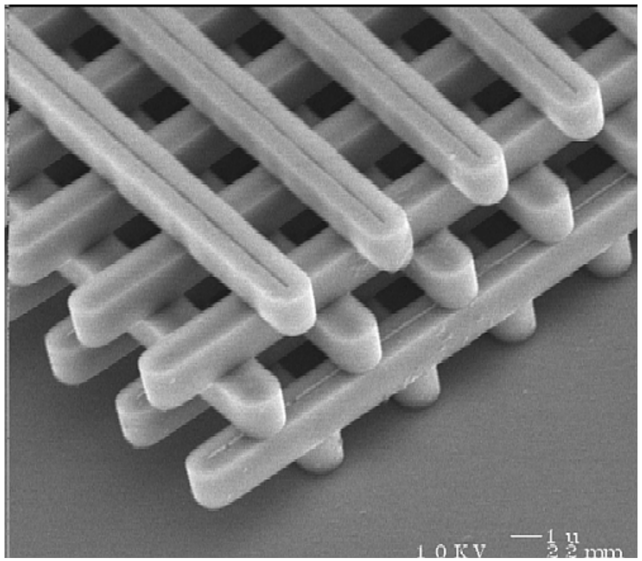
EXAMPLES OF 3D PCs

[Johnson et al., APL. 77]



layer by layer lithography

[Lin et al., JOSA B 18]

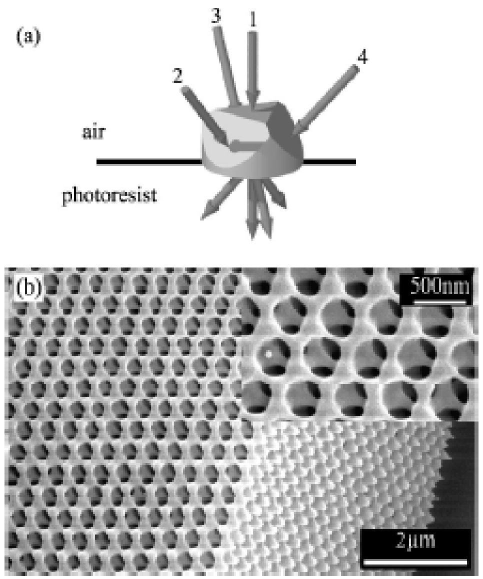


microassembly

Different approaches are developed for 3D PC

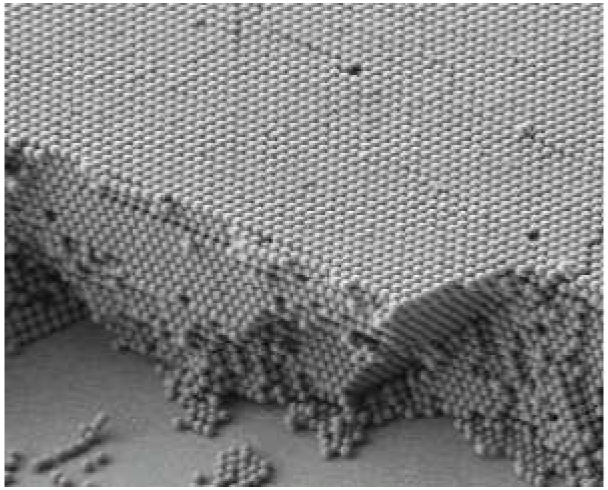
EXAMPLES OF 3D PCs

[Miklyaev et al., APL. 82]



holography

[Vlasov et al., Nature 414]



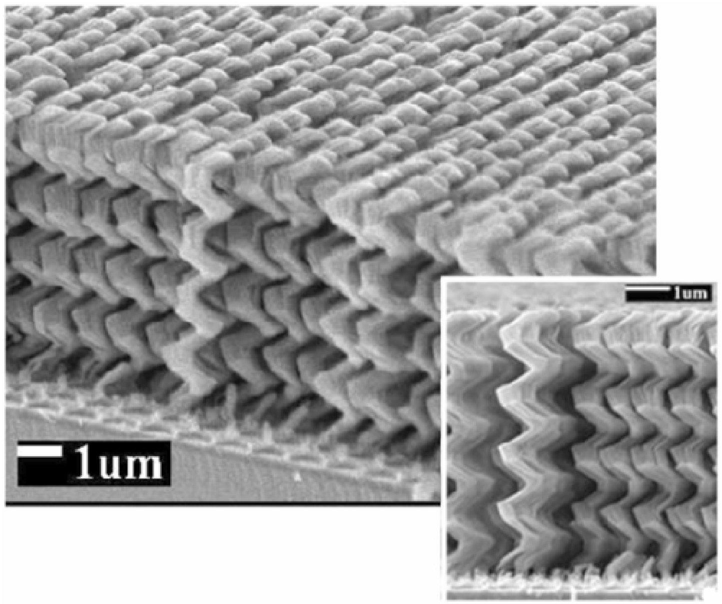
layer by layer lithography

These structures have to be inverted

Different approaches are developed for 3D PC

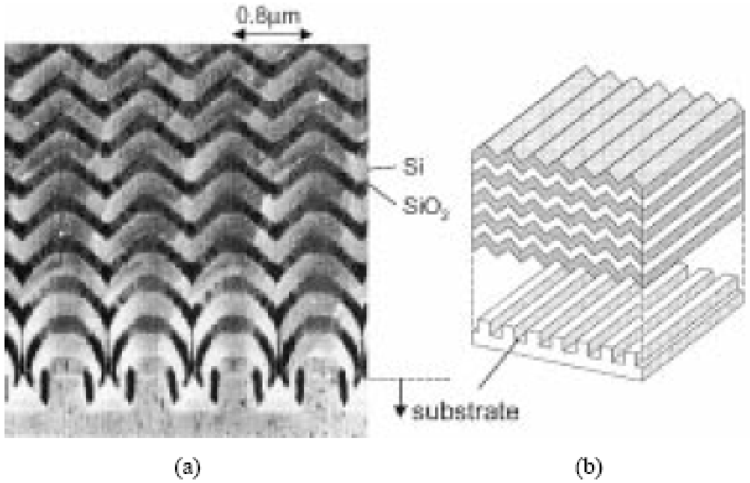
EXAMPLES OF 3D PCs

[S. R. Kennedy et al., NanoLetters 2]



glancing angle deposition (GLAD)

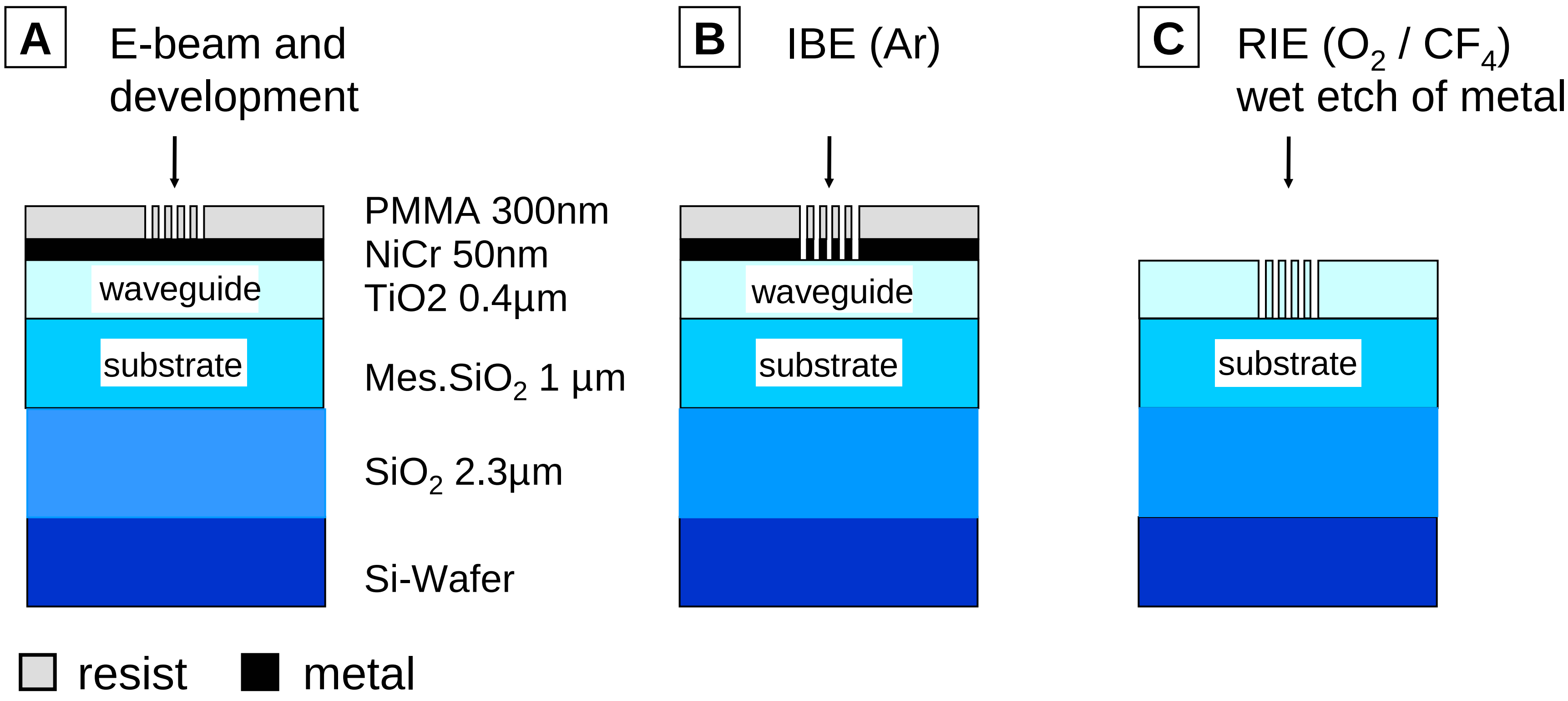
[Kawashima et. al., J.Quant.Electr. 38]



autocloning

2D slab structures are are manufactured using a three step lithography process

TiO$_2$ FABRICATION PROCESS



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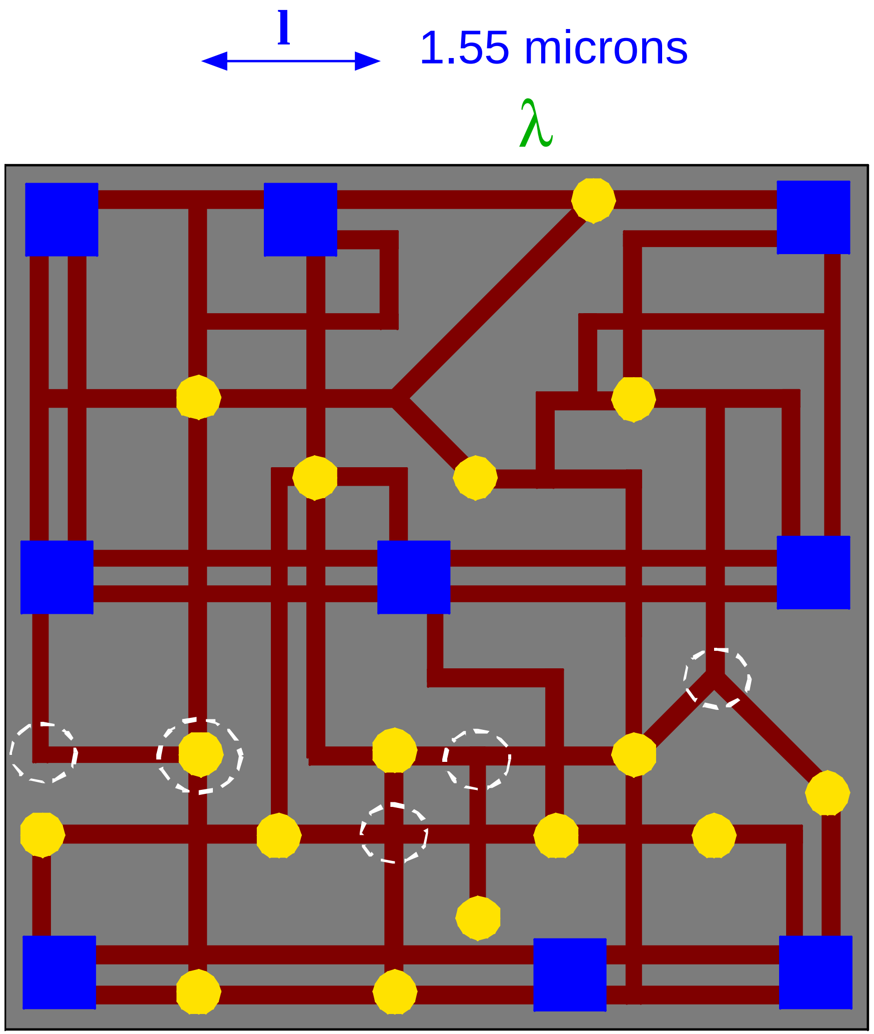
Several applications are discussed in PC community

MOTIVATION FOR PC RESEARCH

- Modification of the density of states
  (Purcell effect, low threshold laser)
- Light guiding around tight corners (ultra compact optics)
- High Q resonators (optical filtering, switching, sensor)
- Refractive optics
- Time delay, dispersion control
- Microwave antenna designs
- Pigments
- PC fibers

Small light circuits can be obtained with PC structures

SCHEMATICAL VIEW OF INTEGRATED DEVICES



Y - splitters

Z - bends

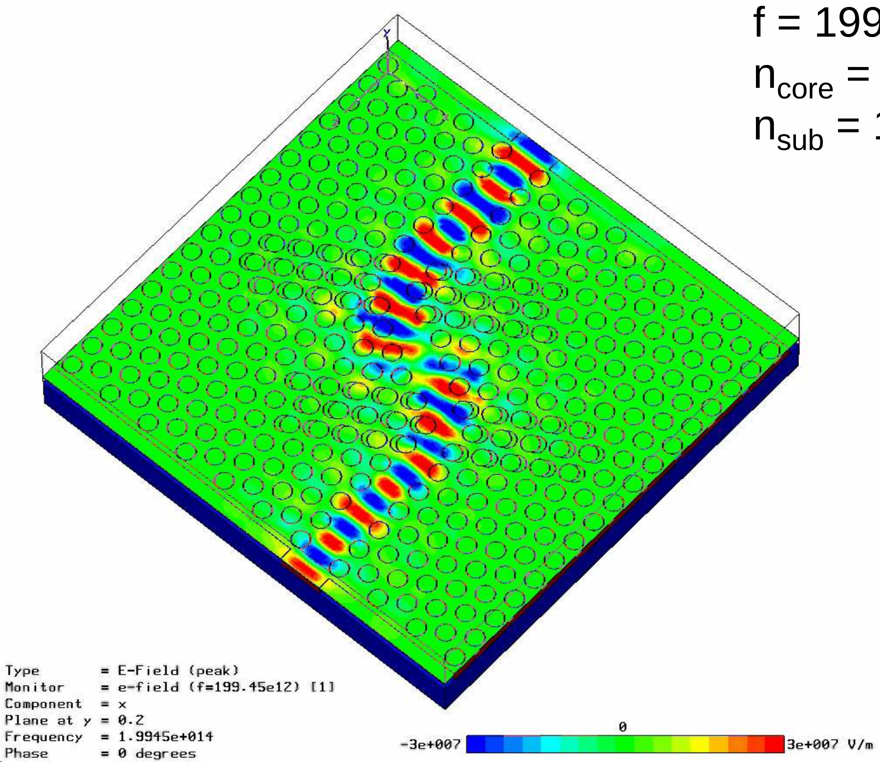
T and X intersections

S. Johnson, tutorial , MIT

Less transmission of comparable channel waveguide

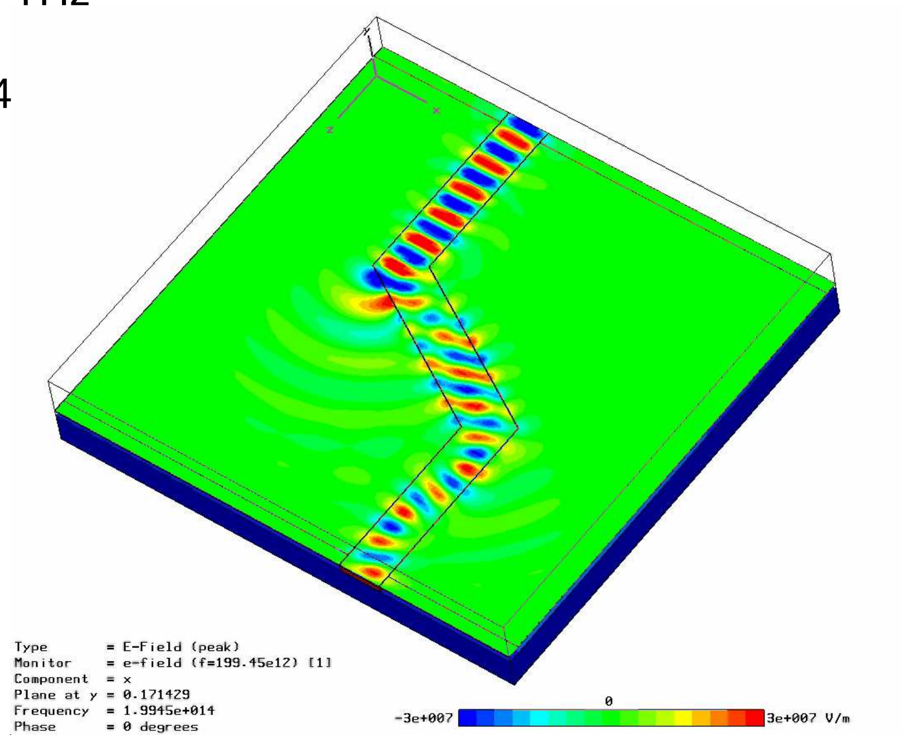
COMPARISON OF PC-BEND VS. BENT CHANNEL WG

PC wg bend



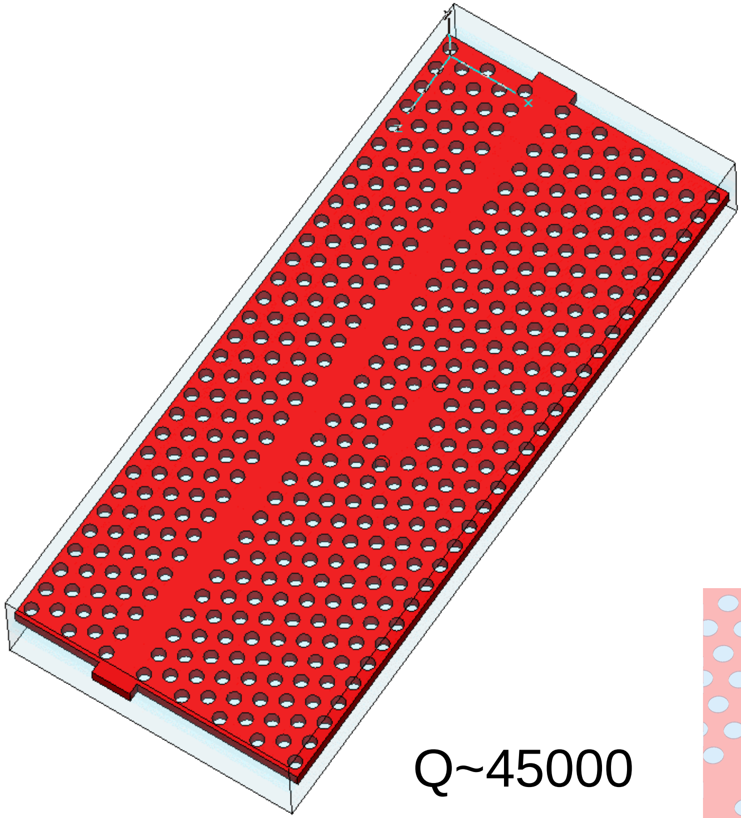
f = 199.45 THz
n_{core} = 2.3
n_{sub} = 1.14

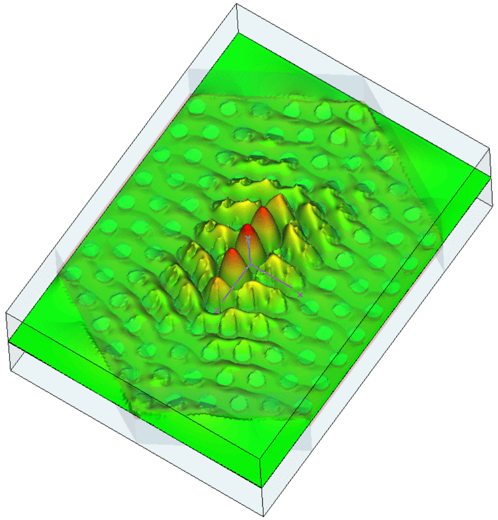
channel wg bend



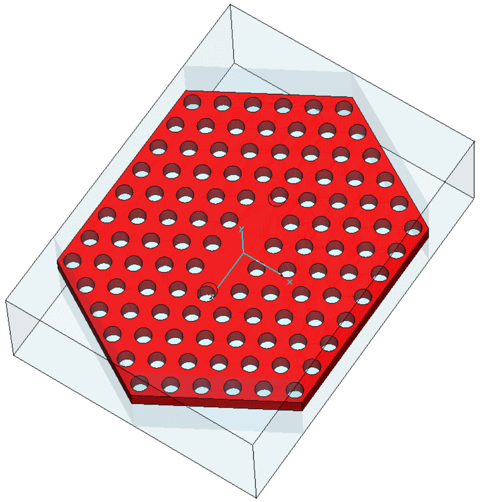
Novel design proposal for tuning drop cavities (Noda)

W1-WG SIDE-COUPLING TO L3-CAVITY EIGENSTATE (n=3.4)

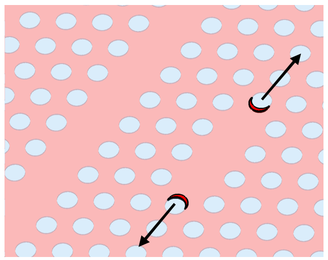




L3 A00/A15 Eigen



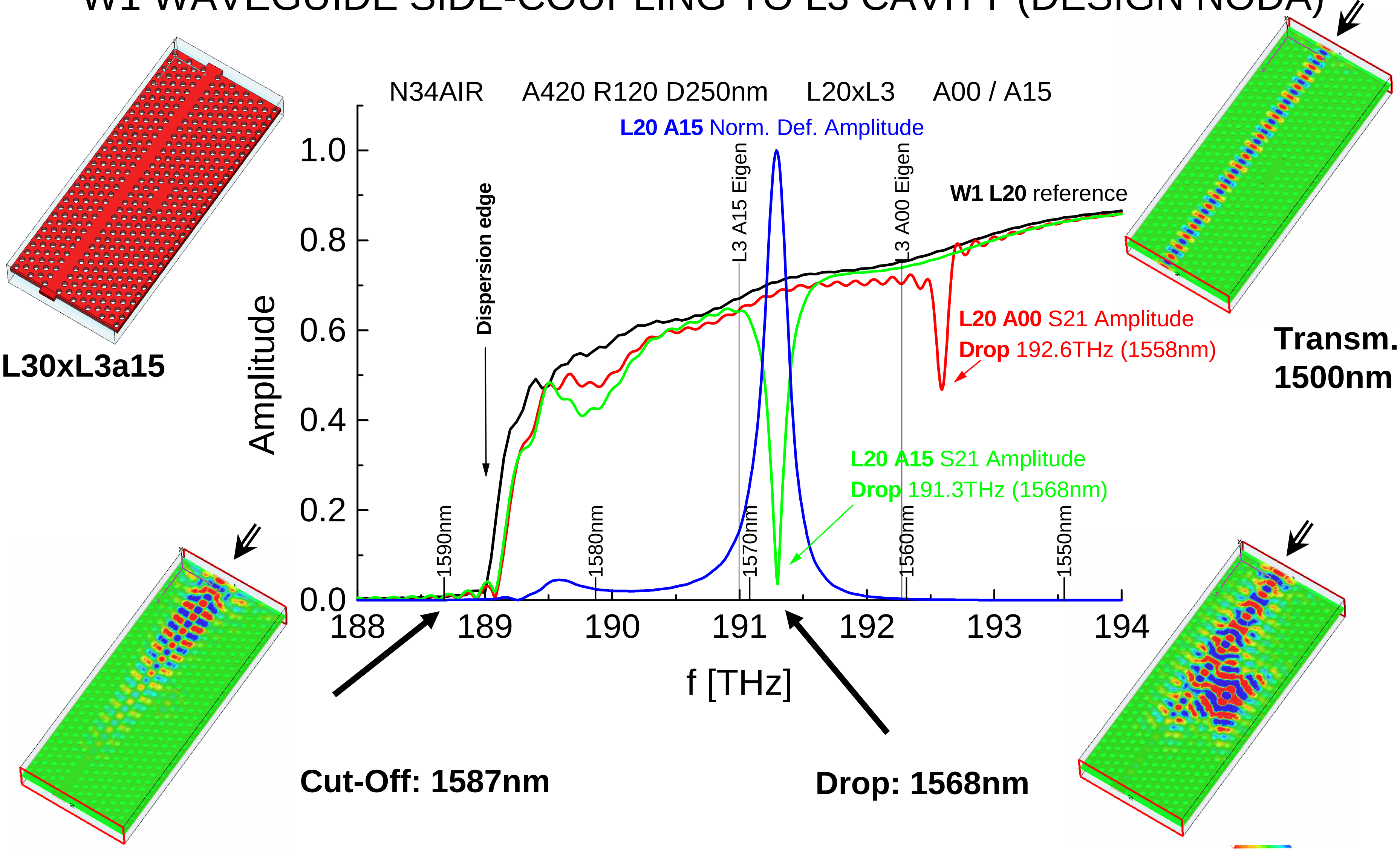
Q~45000



Akahane et. al. Nature 425

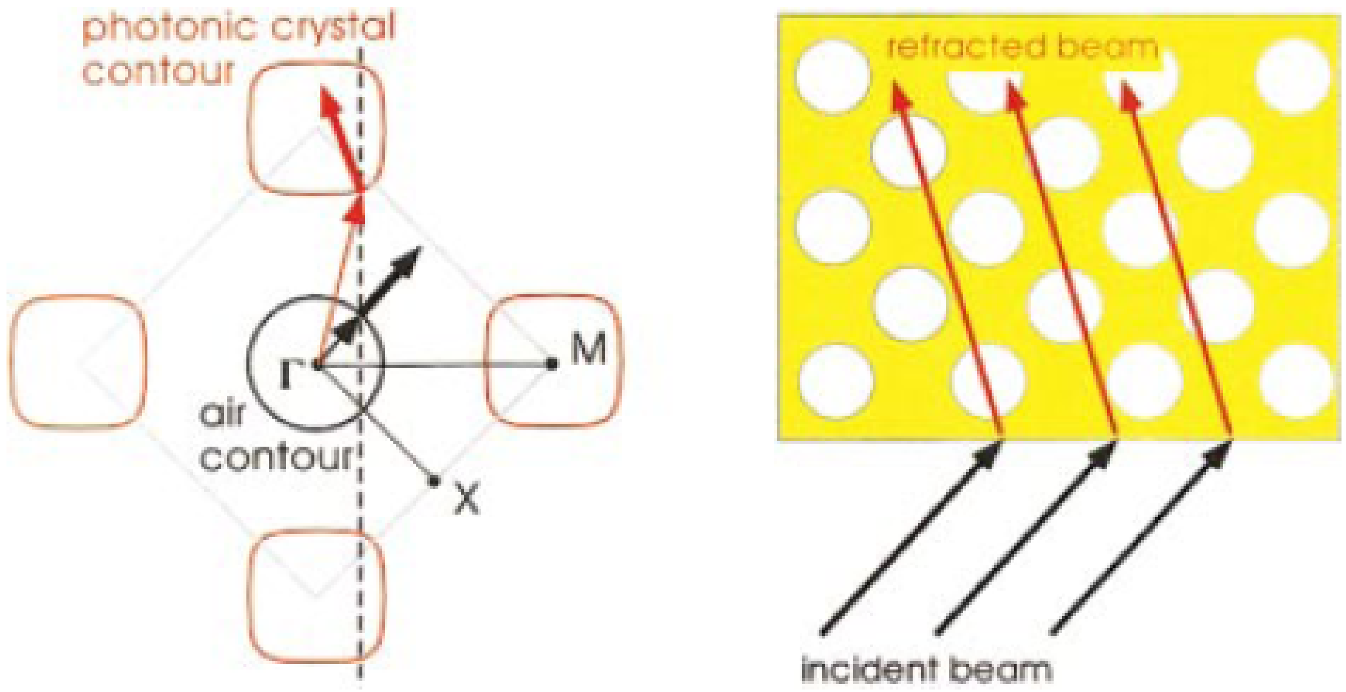
Combination of W1-WG with cavity yields high Q drop

W1 WAVEGUIDE SIDE-COUPLING TO L3 CAVITY (DESIGN NODA)



PC anisotropy can lead to negative refraction of light

WAVEVECTOR DIAGRAM AND BEAM PICTURE

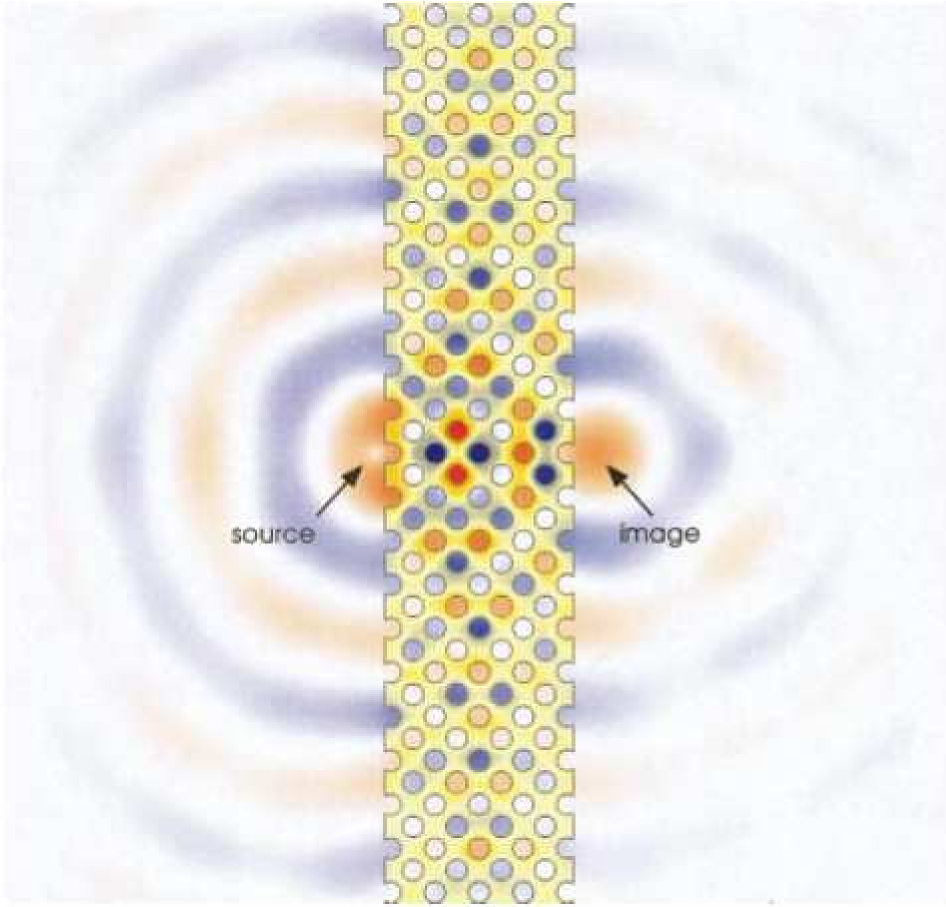


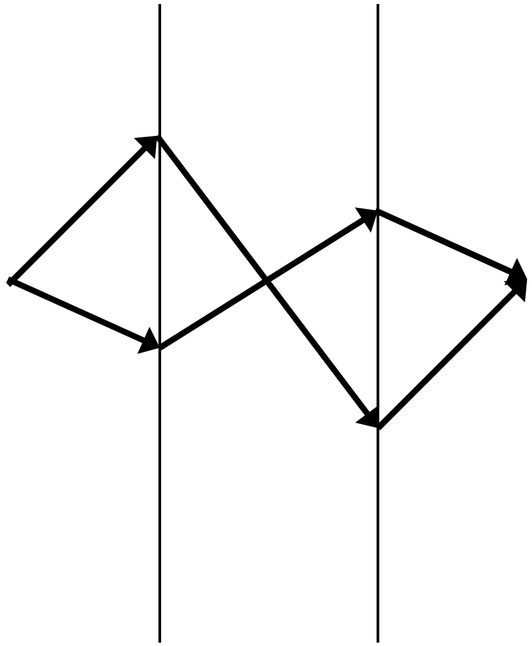
2D square lattice of holes in dielectric

Luo et. al. Phys. Rev.B 65

Negative refraction is a new area of refractive optics

SUPERLENSE

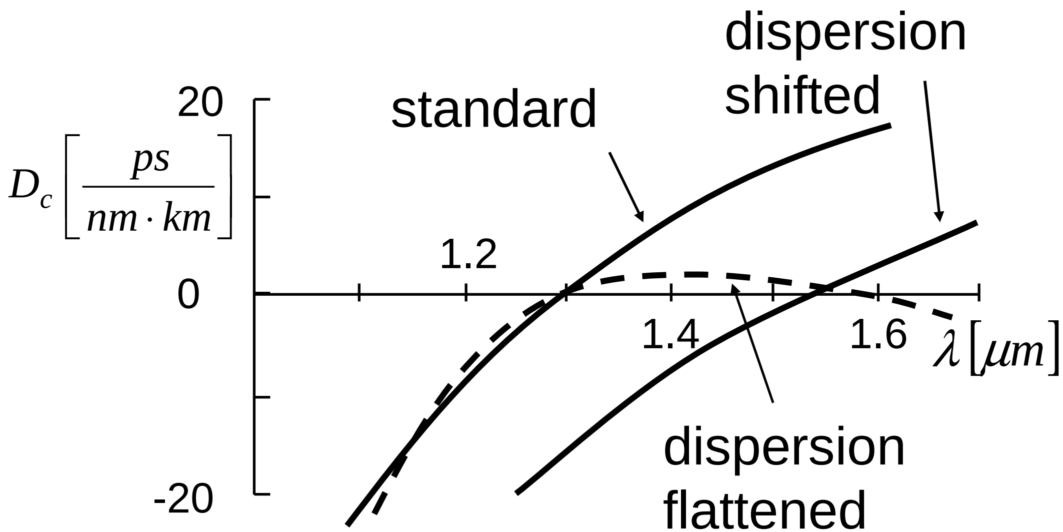


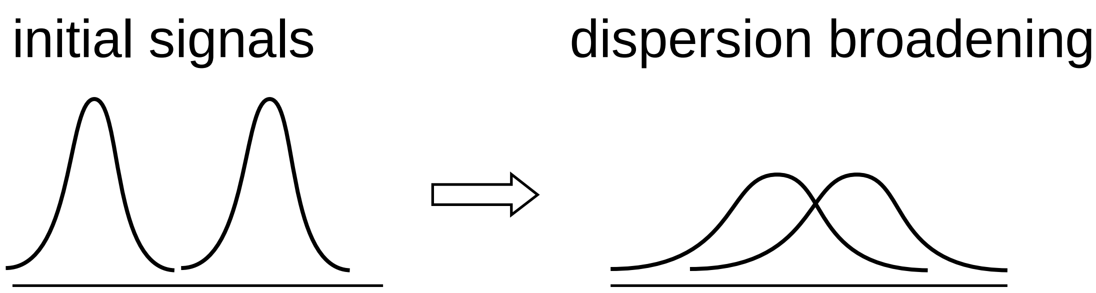


Luo et. al. Phys. Rev.B 65

Dispersion limits the available bandwidth of the fiber

DISPERSION IN OPTICAL FIBERS





dispersion coefficient to account for:

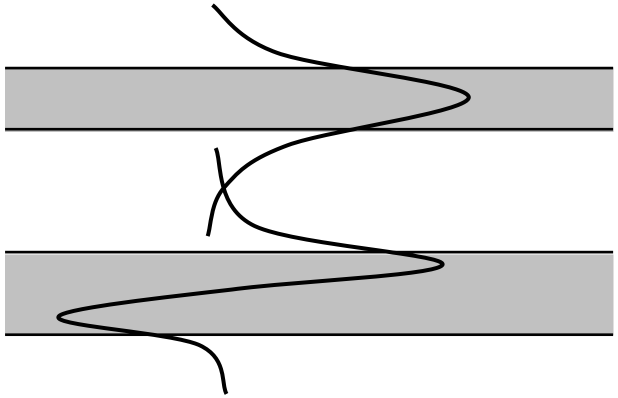
$$D_c \approx \pm 20 \frac{ps}{nm \cdot km}$$

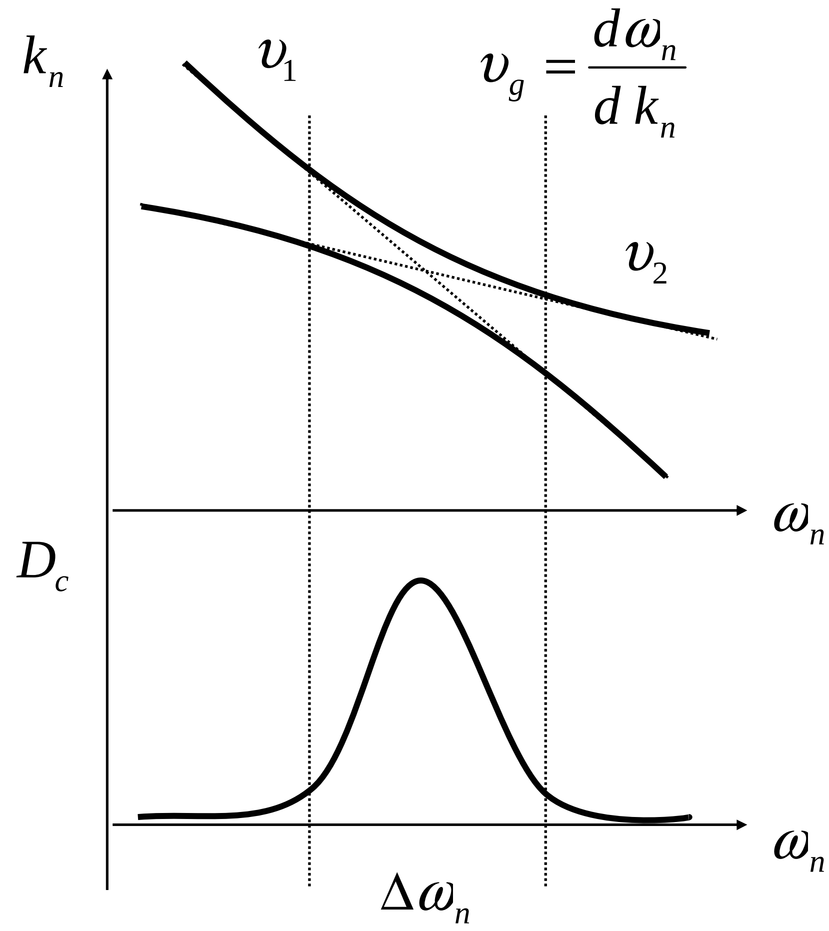
$$L \approx 100\, km$$

$$D \approx \pm 2000 \frac{\mathrm{ps}}{\mathrm{nm}}$$

Coupled modes can be used for dispersion compensation

ANTI-CROSSING POINT OF TWO COUPLED MODES





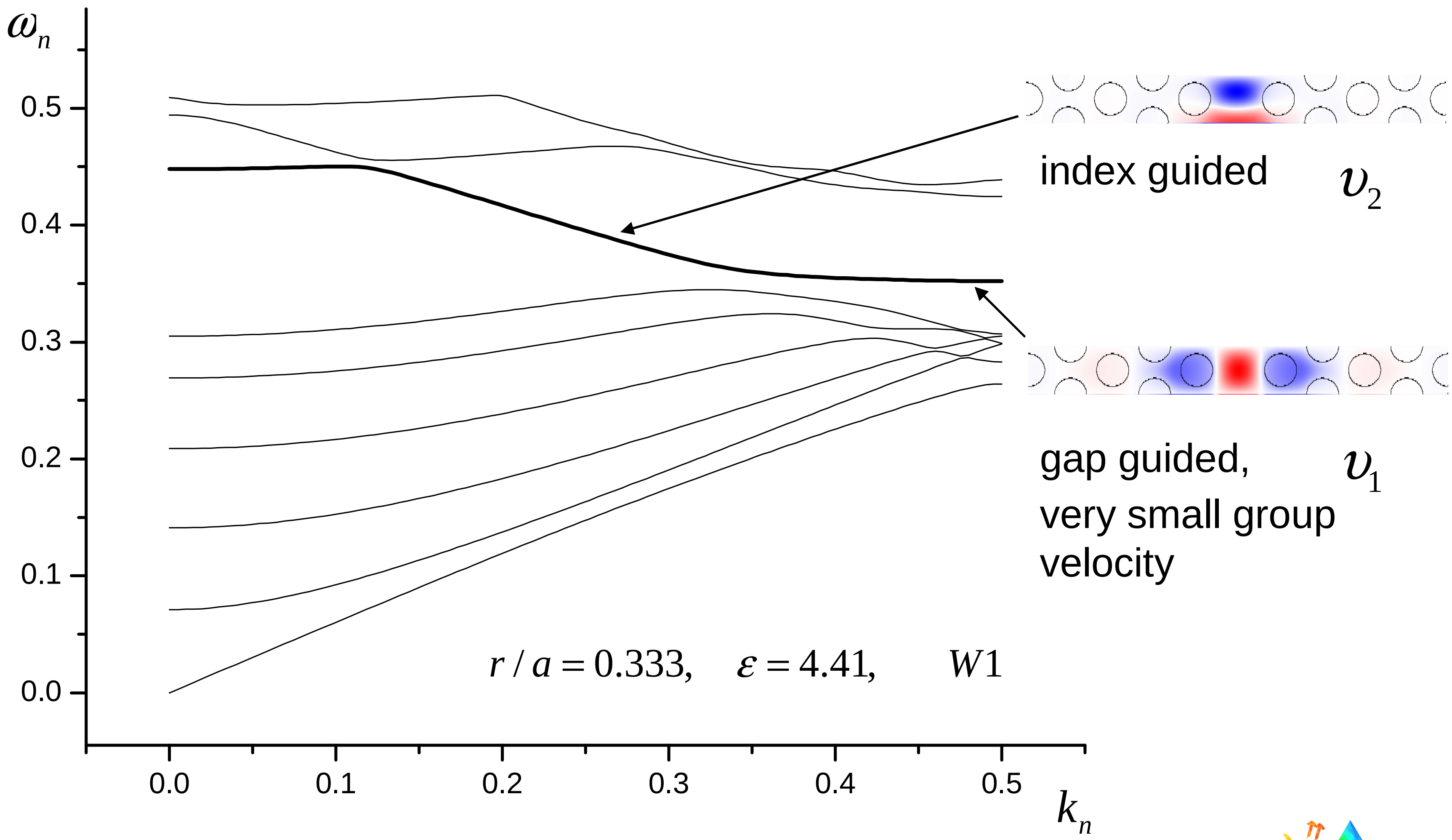
$$D_c = \frac{d(1/\upsilon_g)}{d\lambda} = \frac{1}{\Delta\lambda}\left(\frac{1}{\upsilon_1} - \frac{1}{\upsilon_2}\right)$$

$$\upsilon_{2\max} = c$$

$$\Delta\lambda \downarrow,\ \upsilon_1 \downarrow \quad \Rightarrow \quad D \uparrow$$

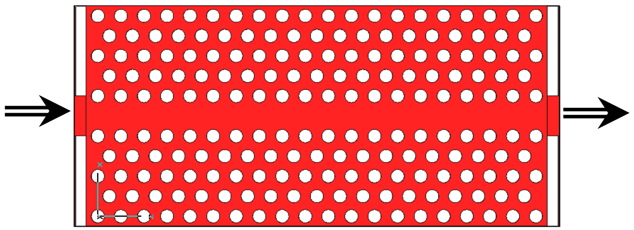
Mode anti-crossing can appear inside PBG region

BAND DIAGRAM OF A PC WAVEGUIDE



Quasi-constant dispersion is achieved in PC waveguides

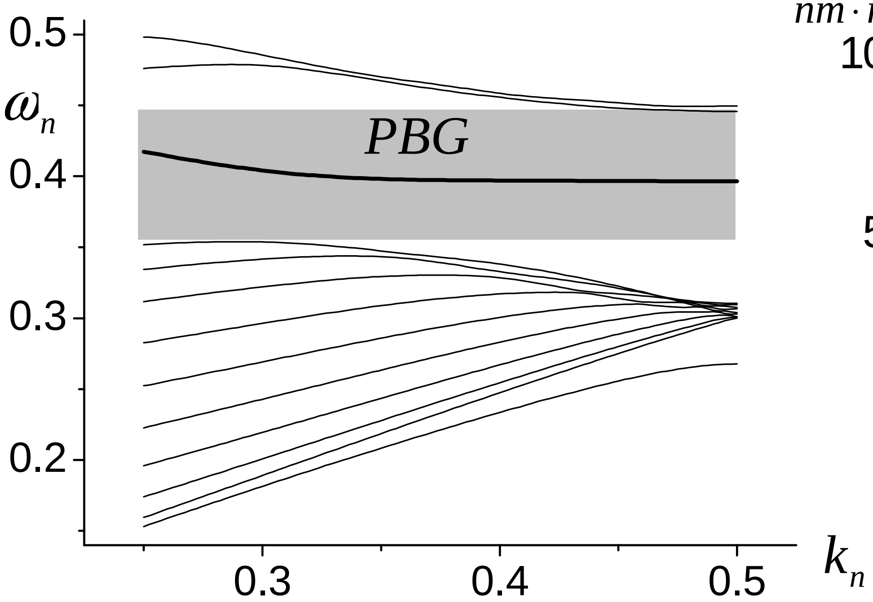
PC WAVEGUIDE, BAND DIAGRAM AND DISPERSION

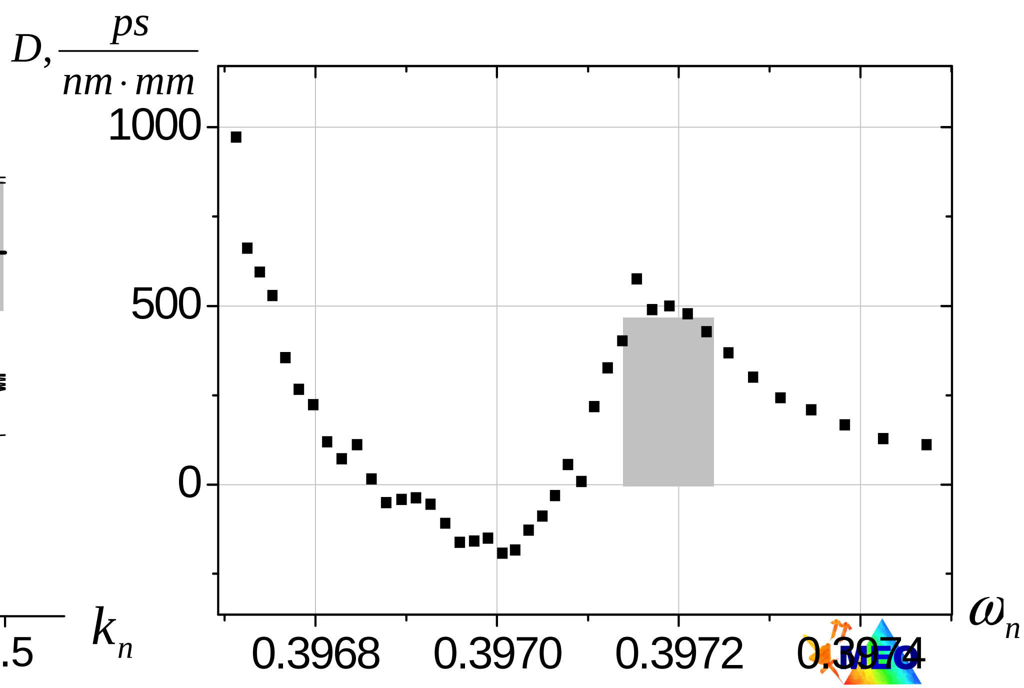


$$r/a = 0.366, \quad \varepsilon = 4.90, \quad W0.8$$

$$\Delta\lambda \approx 1\ channel\ \ (50\,GHz)$$

$$L \approx 5mm$$





Theory of infinite PC structure (band diagram, band gap)

Beam propagation in PC (group velocity direction, Snell's law)

PC as omnidirectional mirror (cavity, waveguides)

2D PC slab structure (light line, losses)

Manufacturing (3D, 2D)

Possible applications (Q-cavities, waveguides, refraction, dispersion)

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